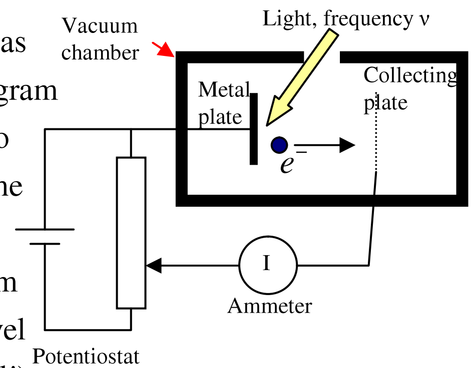
WAVES AND PARTICLES

1. We think of particles as matter highly concentrated in some volume of space, and of waves as being highly spread out. Think of a cricket ball, and of waves in the ocean. The two are completely different! And yet today we are convinced that matter takes the form of waves in some situations and behaves as particles in other situations. This is called wave-particle duality. But do not be afraid - there is no logical contradiction here! In this lecture we shall first consider the evidence that shows the particle nature of light.



2. The photoelectric effect, noted nearly 100 years ago, was crucial for understanding the nature of light. In the diagram shown, when light falls upon a metal plate connected to the cathode of a battery, electrons are knocked out of the plate. They reach a collecting plate that is connected to the battery's anode, and a current is observed. A vacuum is created in the apparatus so that the electrons can travel without hindrance. According to classical (meaning old!) physics we expect the following:

   a)As intensity of light increases, the kinetic energy of the ejected electrons should increase.

   b)Electrons should be emitted for any frequency of light ν, so long as the intensity of the light is sufficiently large.

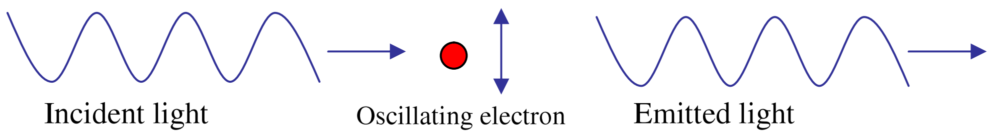
   But the actual observation was completely different and showed the following:

   a)The maximum kinetic energy of the emitted electrons was completely independent of the light intensity, but it did depend on ν.

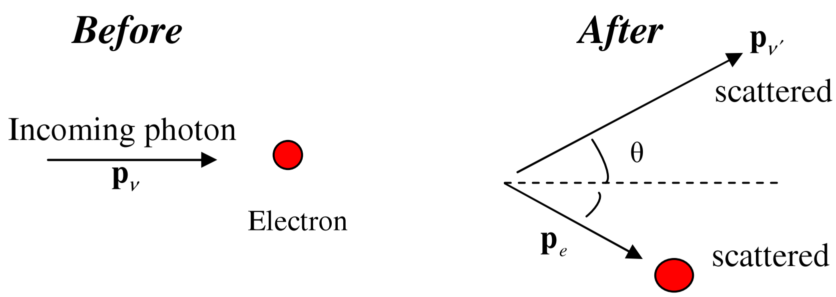
   b)For $\nu < \nu_0$ (i.e. for frequencies below a cut-off frequency) no electrons are emitted no matter how large the light intensity was.

3. In 1905, Einstein realized that the photoelectric effect could be explained if light actually comes in little packets (or quanta) of energy with the energy of each quantum $E = h\nu$. Here h is a universal constant of nature with value $h = 6.63\times10^{-34}$ Joule-seconds, and is known as the Planck Constant. If an electron absorbs a single photon, it would be able to leave the material if the energy of that photon is larger than a certain amount W. W is called the work function and differs from material to material, with a value varying from 2-5 electron volts. The maximum KE of an emitted electron is then $K_{\max} = h\nu - W$. We visualize the photon as a particle for the purposes of this experiment. Note that this is completely different from our earlier understanding that light is a wave!

4. That light is made of photons was confirmed by yet another experiment, carried out by Arthur Compton in 1922. Suppose an electron is placed in the path of a light beam. What will happen? Because light is electromagnetic waves, we expect the electron to oscillate with the same frequency as the frequency of the incident light ν. But because a charged particle radiates em waves, we expect that the electron will also radiate light at frequency ν. So the scattered and incident light have the same ν. But this is not what is observed!

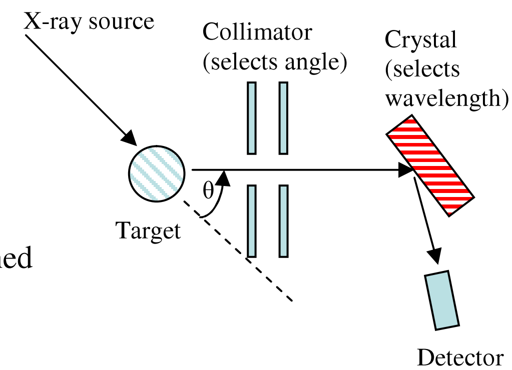


To explain the fact that the scattered light has a different frequency (or wavelength), Compton said that the scattering is a collision between particles of light and electrons. But we know that momentum and energy is conserved in scattering between particles. Specifically, from conservation of energy $h\nu + m_e c^2 = h\nu' + \left(p_e^2 c^2 + m_e^2 c^4\right)^{1/2}$. The last term is the energy of the scattered electron with mass m_e. Next, use the conservation



of momentum. The initial momentum of the photon is entirely along the $\hat{z}$ direction, $\mathbf{p}_\nu = \frac{h}{\lambda}\hat{z} = \mathbf{p}_{\nu'} + \mathbf{p}_e$. By resolving the components and doing a bit of algebra, you can get the change in wavelength $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda_c(1-\cos\theta)$, where the Compton wavelength $\lambda_c = \frac{h}{m_e c} = 2.4\times 10^{-12}$m. Note that $\lambda' - \lambda$ is always positive because $\cos\theta$ has magnitude less than 1. In other words, the frequency of the scattered photon is always less than the frequency of the incoming one. We can understand this result because the incoming photon gives a kick to the (stationary) electron and so it loses energy. Since $E = h\nu$, it follows that the outgoing frequency is decreased. As remarked earlier, it is impossible to understand this from a classical point of view. We shall now see how the Compton effect is actually observed experimentally.

5. In the apparatus shown, X-rays are incident upon a target which contains electrons. Those X-rays which are scattered in a particular angle θ are then selected by the collimator and are incident upon a crystal. The crystal diffracts the X-rays and, as you will recall from the lecture on diffraction, $2d\sin\theta_D = n\lambda$ is the necessary condition. It was thus determined that the changed wavelength of the scattered follows $\lambda' - \lambda = \lambda_c(1-\cos\theta)$.



6. Here is a summary of photon facts:
   a)The relation between the energy and frequency of a photon is $E = h\nu$.
   b)The relativistic formula relating energy and momentum for photons is $E = pc$. Note that in general $E^2 = p^2c^2 + m^2c^4$ for a massive particle. The photon has $m = 0$.
   c)The relation between frequency, wavelength, and photon speed is $\lambda\nu = c$.
   d)From the abbove, the momentum-wavelength relation for photons is $p = \frac{h\nu}{c} = \frac{h}{\lambda}$.
   e)An alternative way of expressing the above is: $E = \hbar\omega$ and $p = \hbar k$. Here $\omega = 2\pi\nu$ and $k = \frac{2\pi}{\lambda}$, $\hbar \equiv \frac{h}{2\pi}$ ($\hbar$ is pronounced h-bar).
   f)Light is always detected as packets (photons); if we look, we never observe half a photon. The number of photons is proportional to the energy density (i.e. to square of the electromagnetic field strength).

7. So light behaves as if made of particles. But do all particles of matter behave as if they are waves? In 1923 a Frenchman, Louis de Broglie, postulated that ordinary matter can have wave-like properties with the wavelength λ related to the particle's momentum p in the same way as for light, $\lambda = \frac{h}{p}$. We shall call λ the de Broglie (pronounced as Deebrolee!) wavelength.

8. Let us estimate some typical De Broglie wavelengths:
   a) The wavelength of 0.5 kg cricket ball moving at 2 m/sec is:

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{0.5\times2} = 6.63\times10^{-34}\ \text{m}$$

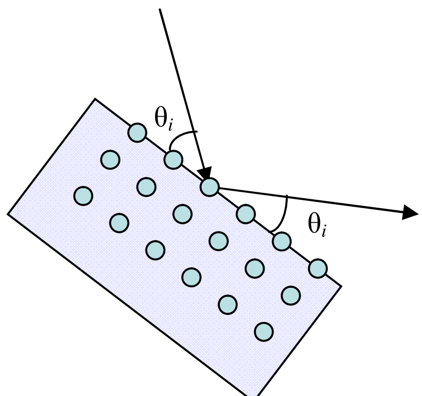
This is extremely small even in comparison to an atom, 10^{-10} m.

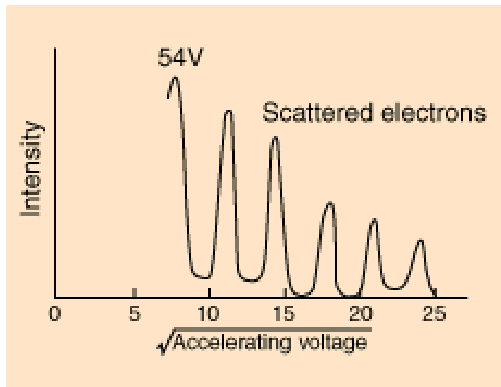
b) The wavelength of an electron with 50eV kinetic energy is calculated from:

$$K = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} \Rightarrow \lambda = \frac{h}{\sqrt{2m_e K}} = 1.7\times10^{-10}\,\text{m}$$

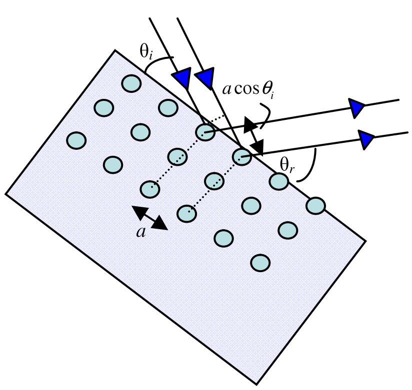
Now you see that we are close to atomic dimensions.

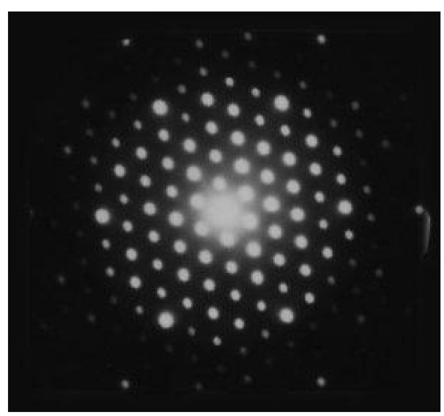
9. If De Broglie's hypothesis is correct, then we can expect that electron waves will undergo interference just like light waves. Indeed, the Davisson-Germer experiment (1927) showed that this was true. At fixed angle, one find sharp peaks in intensity as a function of electron energy. The electron waves hitting the atoms are re-emitted and reflected, and waves from different atoms interfere with each other. One therefore sees the peaks and valleys that are typical of interference (or diffraction) patterns in optical experiments.



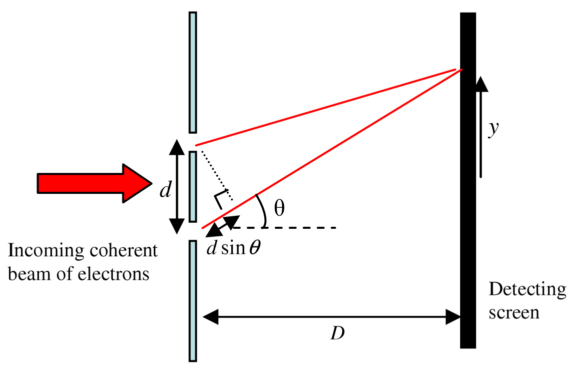


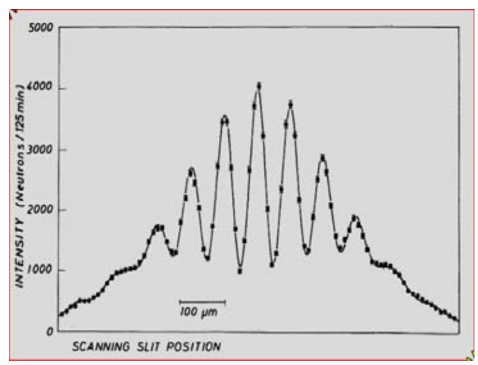
10. Let us look at the interference in some detail. When electrons fall on a crystalline surface, the electron scattering is dominated by surface layers. Note that the identical scattering planes are oriented perpendicular to the surface. Looking at the diagram, we can see that constructive interference happens when $a(\cos\theta_r - \cos\theta_i) = n\lambda$. When this condition is satified, there is a maximum intensity spot. This is actually how we find a and determine the structure of crystals.





11. Let's take a still simpler situation: electrons are incident upon a metal plate with two tiny holes punched into it. The holes - spearated by distance d - are very close together. A screen behind, at distance D, is made of material that flashes whenever it is hit by an electron. A clear interference pattern with peaks and valleys is observed. Let us analyze: there will be a maximum when $d\sin\theta = n\lambda$. If the screen is very far away, i.e. $D >> d$, then θ will be very small and $\sin\theta \approx \theta$. So we then have $\theta \approx \frac{n\lambda}{d}$, and the angular separation between two adjacent minima is $\Delta\theta \approx \frac{\lambda}{d}$. The position on the screen is y, and $y = D\tan\theta \approx D\theta$. So the separation between adjacent maxima is $\Delta y \approx D\Delta\theta$ and hence $\Delta y = \frac{\lambda D}{d}$. This is the separation between two adjacent bright spots. You can see from the experimental data that this is exactly what is observed.





12. The double-slit experiment is so important that we need to discuss it further. Note the following:

a) It doesn't matter whether we use light, electrons, or atoms - they all behave as waves. in this experiment. The wavelength of a matter wave is unconnected to any internal size of particle. Instead it is determined by the momentum, $\lambda = \frac{h}{p}$.

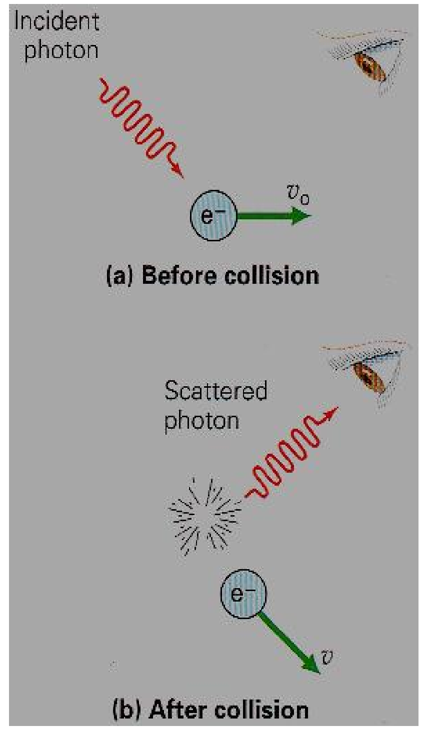
b) If one slit is closed, the interference disappears. So, in fact, each particle goes through both slits at once.

c) The flux of particles arriving at the slits can be reduced so that only one particle arrives at a time. Interference fringes are still observed !Wave-behaviour can be shown by a single atom. In other words, a matter wave can interfere with itself.

d) If we try to find out which slit the particle goes through the interference pattern vanishes! We cannot see the wave/particle nature at the same time.

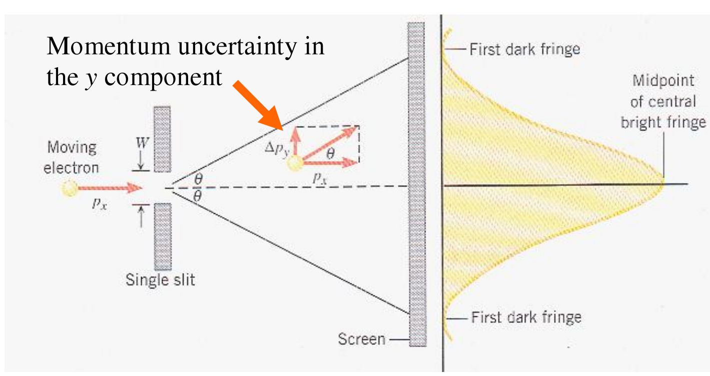
All this is so mysterious and against all our expectations. But that's how Nature is!

13. *Heisenberg Uncertainty Principle.* In real life we are perfectly familiar with seeing a cricket ball at rest - it has both a fixed position and fixed (zero) momentum. But in the microscopic world (atoms, nuclei, quarks, and still smaller distance scales) this is impossible. Heisenberg, the great German physicist, pointed out that if we want to see an electron, then we have to hit it with some other particle. So let's say that a photon hits an electron and then enters a detector. It will carry information to the detector of the position and velocity of the electron, but in doing so it will have changed the momentum of the electron. So if the electron was initially at rest, it will no longer be so afterwards. The act of measurement changes the state of the system! This is true no matter how you do the experiment. The statement of the principle is:



"If the position of a particle can be fixed with accuracy Δx, then the maximum accuracy with which the momentum can be fixed is Δp, where $\Delta x \Delta p \geq \hbar/2$." We call Δx the position uncertainty, and Δp the momentum uncertainty. Note that their product is fixed. Therefore, if we fix the position of the particle (make Δx very small), then the particle will move about randomly very fast (and have Δp very large).

14. By using the De Broglie hypothesis in a simple gedanken experiment, we can see how the uncertainty principle emerges. Electrons are incident upon a single slit and strike a screen far away. The first dark fringe will be when $W \sin\theta = \lambda$. Since θ is small, we can use $\sin\theta \approx \theta$, and so $\theta \approx \dfrac{\lambda}{W}$. But, on the other hand, $\tan\theta = \dfrac{\Delta p_y}{p_x}$ and $\theta \approx \dfrac{\Delta p_y}{p_x}$. So, $\dfrac{\Delta p_y}{h/\lambda} = \dfrac{\lambda}{W}$. But W is really the uncertainty in the y position and we should call it Δy.



Thus we have found that $\Delta p_y \Delta y \approx h$. The important point here is that by localizing the y position of the electron to the width of the slit, we have forced the electron to acquire a momentum in the y direction whose uncertainty is Δp_y.

15. In a proper course in quantum mechanics, one can give a definite mathematical meaning to Δx and Δp_x etc, and derive the uncertainty relations:

$$\Delta x \Delta p_x \geq \hbar/2, \quad \Delta y \Delta p_y \geq \hbar/2, \quad \Delta z \Delta p_z \geq \hbar/2$$

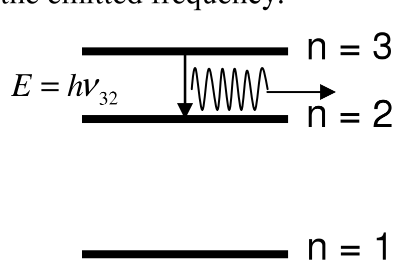
Note the following:

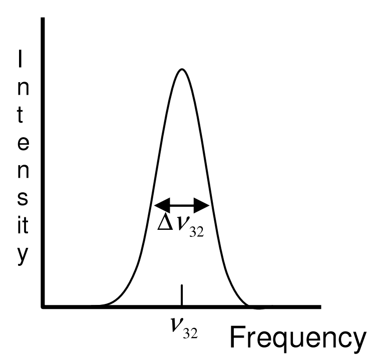
a) There no uncertainty principle for the product $\Delta x \Delta p_y$. In other words, we can know in principle the position in one direction precisely together with the momentum in another direction.

b) The thought experiment I discussed seems to imply that, while prior to experiment we have well defined values, it is the act of measurement which introduces the uncertainty by disturbing the particle's position and momentum. Nowadays it is more widely accepted that quantum uncertainty (lack of determinism) is intrinsic to the theory and does not come about just because of the act of measurement.

16. There is also an Energy-Time Uncertainty Principle which states that $\Delta E \Delta t \geq \hbar/2$. This says that the principle of energy conservation can be violated by amount ΔE, but only for a short time given by Δt. The quantity ΔE is called the uncertainty in the energy of a system.

17. One consequence of $\Delta E \Delta t \geq \hbar/2$ is that the level of an atom does not have an exact value. So, transitions between energy levels of atoms are never perfectly sharp in frequency. So, for example, as shown below an electron in the $n = 3$ state will decay to a lower level after a lifetime of order $t \approx 10^{-8}$s. There is a corresponding "spread" in the emitted frequency.





QUESTIONS AND EXERCISES

Q.1 The highest wavelength for which photoelectrons are emitted from tungsten is 2300 A^o. Determine the energy of an electron ejected from the metal's surface by ultraviolet light of wavelength 1900 A^o.

Q.2 Photoelectrons emitted from a cesium plate illuminated with ultraviolet light of wave-length 2000 A^o are stopped by a potential of 4.21V. Find the work function of cesium.

Q.3 a) Show that the De Broglie wavelength of an electron with kinetic energy E (eV) is

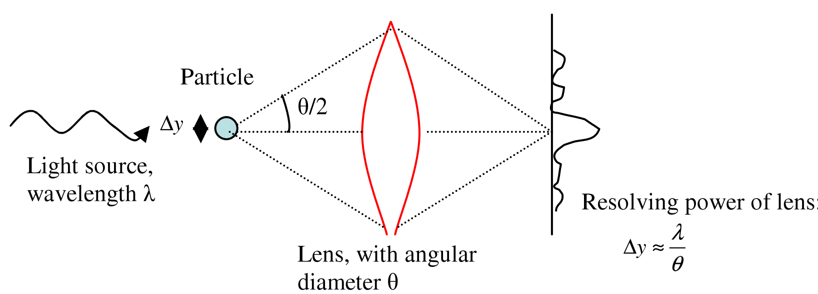
$$\lambda_e = \frac{12.3\times10^{-8}}{E^{1/2}} \text{ cm.}$$

b)Show that the De Broglie wavelength of a proton with kinetic energy E (eV) is

$$\lambda_p = \frac{0.29\times10^{-8}}{E^{1/2}} \text{ cm.}$$

Q.4 Complete the steps in point 4 (Compton Effect) to show that $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$.

Q.5 The Heisenberg microscope is an imaginary device to measure the position (y) and momentum (p) of a particle.



Photons transfer momentum to the particle when they scatter.

a) Show that the magnitude of p is the same before and after the collision.

b) Uncertainty in photon y-momentum= Uncertainty in particle y-momentum. Why?

c) Show that p_y lies in the range $-p\sin(\theta/2) \le p_y \le p\sin(\theta/2)$. Thus, the uncertainty $\Delta p_y = 2p\sin(\theta/2) \approx p\theta$.

d) Use the de Broglie relation $p = h/\lambda$ to show that $\Delta p_y \approx \frac{h\theta}{\lambda}$. Then use $\Delta y \approx \frac{\lambda}{\theta}$ to get back $\Delta p_y \Delta y \approx h$.