Tutorial #7

Q1) $|\psi\rangle = i|\varphi_1\rangle + 3i|\varphi_2\rangle - |\varphi_3\rangle$

$|\chi\rangle = |\varphi_1\rangle - i|\varphi_2\rangle + 5i|\varphi_3\rangle$

and $\langle\varphi_i|\varphi_j\rangle = \delta_{ij}$

(a) $\langle\psi|\psi\rangle = (-i)(i) + (-3i)(3i) + (-1)(-1)$

$= 1 + 9 + 1$

$= 11$

$\langle\chi|\chi\rangle = (1)(1) + (+i)(-i) + (-5i)(5i)$

$= 1 + 1 + 25$

$= 27$

$\langle\psi|\chi\rangle = (-i)(1) + (-3i)(-i) + (-1)(5i)$

$= -i - 3 - 5i$

$= -3 - 6i$

$\langle\chi|\psi\rangle = (1)(+i) + (i)(3i) + (-5i)(-1)$

$= i + 5i - 3$

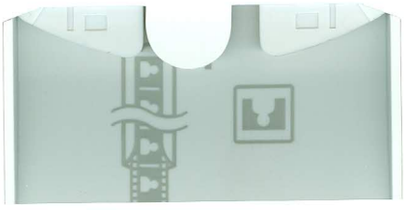
$= -3 + 6i$

$\langle\psi+\chi|\psi+\chi\rangle = (\langle\psi| + \langle\chi|)(|\psi\rangle + |\chi\rangle)$

$= \langle\psi|\psi\rangle + \langle\chi|\psi\rangle + \langle\psi|\chi\rangle + \langle\chi|\chi\rangle$

$= 11 + 27 - 3 - 6i - 3 + 6i$

$= 32$



(b) $|\psi\rangle\langle\chi| = \left(i|\varphi_1\rangle + 3i|\varphi_2\rangle - |\varphi_3\rangle\right)\left(\langle\varphi_1| + i\langle\varphi_2| - 5i\langle\varphi_3|\right)$

$$= i\,|\varphi_1\rangle\langle\varphi_1| - |\varphi_1\rangle\langle\varphi_2|$$
$$= \left(i|\varphi_1\rangle + 3i|\varphi_2\rangle - |\varphi_3\rangle\right)\langle\varphi_1| - \left(|\varphi_1\rangle + 3|\varphi_2\rangle + i|\varphi_3\rangle\right)\langle\varphi_2|$$
$$+ 5\left(|\varphi_1\rangle + 3|\varphi_2\rangle + i|\varphi_3\rangle\right)\langle\varphi_3|$$

$$|\chi\rangle\langle\psi| = \left(|\varphi_1\rangle - i|\varphi_2\rangle + 5i|\varphi_3\rangle\right)\left(-i\langle\varphi_1| - 3i\langle\varphi_2| - \langle\varphi_3|\right)$$
$$= \left(-i|\varphi_1\rangle - |\varphi_2\rangle + 5|\varphi_3\rangle\right)\langle\varphi_1| + \left(-3i|\varphi_1\rangle - 3|\varphi_2\rangle + 15|\varphi_3\rangle\right)\langle\varphi_2|$$
$$- \left(|\varphi_1\rangle - i|\varphi_2\rangle + 5i|\varphi_3\rangle\right)\langle\varphi_3|$$

They are not equal.

(c) $|\psi\rangle^\dagger = \langle\psi| = -i\langle\varphi_1| - 3i\langle\varphi_2| - \langle\varphi_3|$

$|\chi\rangle^\dagger = \langle\chi| = \langle\varphi_1| + i\langle\varphi_2| - 5i\langle\varphi_3|$

$(|\psi\rangle\langle\chi|)^\dagger = |\chi\rangle\langle\psi|$

$(|\chi\rangle\langle\psi|)^\dagger = |\psi\rangle\langle\chi|$

Q2) $|\psi_1\rangle = |\varphi_1\rangle + 4i|\varphi_2\rangle + 5|\varphi_3\rangle$

$|\psi_2\rangle = b|\varphi_1\rangle + 4|\varphi_2\rangle - 3i|\varphi_3\rangle$

and $\langle\varphi_i|\varphi_j\rangle = \delta_{ij}$

$$\langle\psi_1|\psi_2\rangle = \left(\langle\varphi_1| - 4i\langle\varphi_2| + 5\langle\varphi_3|\right)\left(b|\varphi_1\rangle + 4|\varphi_2\rangle - 3i|\varphi_3\rangle\right)$$
$$= b\langle\varphi_1|\varphi_1\rangle - 16i\langle\varphi_2|\varphi_2\rangle - 15i\langle\varphi_3|\varphi_3\rangle$$
$$= b - 31i$$