

## TUTORIAL # 7

$$\begin{aligned} \textcircled{1)} \quad |\psi\rangle &= i|\varphi_1\rangle + 3i|\varphi_2\rangle - |\varphi_3\rangle \\ |\chi\rangle &= |\varphi_1\rangle - i|\varphi_2\rangle + 5i|\varphi_3\rangle \\ \text{and } \langle\varphi_i|\varphi_j\rangle &= \delta_{ij} \end{aligned}$$

~~(a)  $\langle\psi|\psi\rangle = (\langle\varphi_1| + i\langle\varphi_2| - \langle\varphi_3|)(i|\varphi_1\rangle + 3i|\varphi_2\rangle - |\varphi_3\rangle)$~~

$$\begin{aligned} \text{(a)} \quad \langle\psi|\psi\rangle &= (-i)(i) + (-3i)(3i) + (-1)(-1) \\ &= 1 + 9 + 1 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \langle\chi|\chi\rangle &= (1)(1) + (+i)(-i) + (-5i)(5i) \\ &= 1 + 1 + 25 \\ &= 27 \end{aligned}$$

$$\begin{aligned} \langle\psi|\chi\rangle &= (-i)(1) + (-3i)(-i) + (-1)(5i) \\ &= -i - 3 - 5i \\ &= -3 - 6i \end{aligned}$$

$$\begin{aligned} \langle\chi|\psi\rangle &= (1)(+i) + (i)(3i) + (-5i)(-1) \\ &= i + 5i - 3 \\ &= -3 + 6i \end{aligned}$$

$$\begin{aligned} \langle\psi+\chi|\psi+\chi\rangle &= (\langle\psi| + \langle\chi|)(|\psi\rangle + |\chi\rangle) \\ &= \langle\psi|\psi\rangle + \langle\chi|\psi\rangle + \langle\psi|\chi\rangle + \langle\chi|\chi\rangle \\ &= 11 + 27 - 3 - 6i - 3 + 6i \\ &= 32 \end{aligned}$$



$$(b) \quad |\psi\rangle\langle\chi| = (i|\varphi_1\rangle + 3i|\varphi_2\rangle - |\varphi_3\rangle)(\langle\varphi_1| + i\langle\varphi_2| - 5i\langle\varphi_3|)$$

$$= i|\varphi_1\rangle\langle\varphi_1| - |\varphi_1\rangle\langle\varphi_2|$$

$$= (i|\varphi_1\rangle + 3i|\varphi_2\rangle - |\varphi_3\rangle)\langle\varphi_1| - (|\varphi_1\rangle + 3|\varphi_2\rangle + i|\varphi_3\rangle)\langle\varphi_2| + 5(|\varphi_1\rangle + 3|\varphi_2\rangle + i|\varphi_3\rangle)\langle\varphi_3|$$

$$|\chi\rangle\langle\psi| = (|\varphi_1\rangle - i|\varphi_2\rangle + 5i|\varphi_3\rangle)(-i\langle\varphi_1| - 3i\langle\varphi_2| - \langle\varphi_3|)$$

$$= (-i|\varphi_1\rangle - |\varphi_2\rangle + 5|\varphi_3\rangle)\langle\varphi_1| + (-3i|\varphi_1\rangle - 3|\varphi_2\rangle + 15|\varphi_3\rangle)\langle\varphi_2| - (|\varphi_1\rangle - i|\varphi_2\rangle + 5i|\varphi_3\rangle)\langle\varphi_3|$$

They are not equal.

$$(c) \quad |\psi\rangle^\dagger = \langle\psi| = -i\langle\varphi_1| - 3i\langle\varphi_2| - \langle\varphi_3|$$

$$|\chi\rangle^\dagger = \langle\chi| = \langle\varphi_1| + i\langle\varphi_2| - 5i\langle\varphi_3|$$

$$(|\psi\rangle\langle\chi|)^\dagger = |\chi\rangle\langle\psi|$$

$$(|\chi\rangle\langle\psi|)^\dagger = |\psi\rangle\langle\chi|$$

$$(Q2) \quad |\psi_1\rangle = |\varphi_1\rangle + 4i|\varphi_2\rangle + 5|\varphi_3\rangle$$

$$|\psi_2\rangle = b|\varphi_1\rangle + 4|\varphi_2\rangle - 3i|\varphi_3\rangle$$

$$\text{and } \langle\varphi_i|\varphi_j\rangle = \delta_{ij}$$

$$\langle\psi_1|\psi_2\rangle = (\langle\varphi_1| - 4i\langle\varphi_2| + 5\langle\varphi_3|)(b|\varphi_1\rangle + 4|\varphi_2\rangle - 3i|\varphi_3\rangle)$$

$$= b\langle\varphi_1|\varphi_1\rangle - 16i\langle\varphi_2|\varphi_2\rangle - 15i\langle\varphi_3|\varphi_3\rangle$$

$$= b - 31i$$



