

TUTORIAL # 6

$$\begin{aligned} \text{(Q1)} \quad |\psi\rangle &= 9i|\varphi_1\rangle + 2|\varphi_2\rangle \\ |\chi\rangle &= \frac{-i}{\sqrt{2}}|\varphi_1\rangle + \frac{1}{\sqrt{2}}|\varphi_2\rangle \end{aligned}$$

$$\text{(a)} \quad \langle\chi| = \frac{i}{\sqrt{2}}\langle\varphi_1| + \frac{1}{\sqrt{2}}\langle\varphi_2|$$

$$\begin{aligned} \bullet \quad |\psi\rangle\langle\chi| &= (9i|\varphi_1\rangle + 2|\varphi_2\rangle) \left(\frac{i}{\sqrt{2}}\langle\varphi_1| + \frac{1}{\sqrt{2}}\langle\varphi_2| \right) \\ &= \frac{-9}{\sqrt{2}}|\varphi_1\rangle\langle\varphi_1| + \sqrt{2}i|\varphi_2\rangle\langle\varphi_1| + \frac{9i}{\sqrt{2}}|\varphi_1\rangle\langle\varphi_2| \\ &\quad + \sqrt{2}|\varphi_2\rangle\langle\varphi_2| \end{aligned}$$

$$\begin{aligned} \bullet \quad |\chi\rangle\langle\psi| &= \left(\frac{-i}{\sqrt{2}}|\varphi_1\rangle + \frac{1}{\sqrt{2}}|\varphi_2\rangle \right) (-9i\langle\varphi_1| + 2\langle\varphi_2|) \\ &= \frac{-9}{\sqrt{2}}|\varphi_1\rangle\langle\varphi_1| - \frac{9i}{\sqrt{2}}|\varphi_2\rangle\langle\varphi_1| \\ &\quad - i\sqrt{2}|\varphi_1\rangle\langle\varphi_2| + \sqrt{2}|\varphi_2\rangle\langle\varphi_2| \end{aligned}$$

$$\text{(b)} \quad (|\psi\rangle)^\dagger = \langle\psi| = -9i\langle\varphi_1| + 2\langle\varphi_2|$$

$$(|\chi\rangle)^\dagger = \langle\chi| = \frac{i}{\sqrt{2}}\langle\varphi_1| + \frac{1}{\sqrt{2}}\langle\varphi_2|$$

$$(|\psi\rangle\langle\chi|)^{\dagger} = |\chi\rangle\langle\psi| = \frac{-9}{\sqrt{2}} |\varphi_1\rangle\langle\varphi_1| - \frac{9i}{\sqrt{2}} |\varphi_2\rangle\langle\varphi_1| \\ - \sqrt{2}i |\varphi_1\rangle\langle\varphi_2| + \sqrt{2} |\varphi_2\rangle\langle\varphi_2|$$

$$(|\chi\rangle\langle\psi|)^{\dagger} = |\psi\rangle\langle\chi| = \frac{-9}{\sqrt{2}} |\varphi_1\rangle\langle\varphi_1| + \frac{9i}{\sqrt{2}} |\varphi_1\rangle\langle\varphi_2| \\ + \sqrt{2}i |\varphi_2\rangle\langle\varphi_1| + \sqrt{2} |\varphi_2\rangle\langle\varphi_2|$$

Q2)

$$\hat{B}|\varphi_1\rangle = |\varphi_1\rangle$$

$$\hat{B}|\varphi_2\rangle = 4|\varphi_2\rangle$$

$$\hat{B}|\varphi_3\rangle = 9|\varphi_3\rangle$$

$$\langle B \rangle = \langle \psi | \hat{B} | \psi \rangle$$

$$\hat{B}|\psi\rangle = \frac{1}{\sqrt{2}} |\varphi_1\rangle - 4\frac{\sqrt{2}}{\sqrt{5}} |\varphi_2\rangle + 9\frac{1}{\sqrt{10}} |\varphi_3\rangle$$

$$\langle \psi | \hat{B} | \psi \rangle = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{\sqrt{2}}{\sqrt{5}}\right)\left(-4\frac{\sqrt{2}}{\sqrt{5}}\right) + \left(\frac{1}{\sqrt{10}}\right)\left(\frac{9 \times 1}{\sqrt{10}}\right) \\ = \frac{1}{2} + \frac{8}{5} + \frac{9}{10} = 3$$

$$\therefore \langle B \rangle = \langle \psi | \hat{B} | \psi \rangle = 3$$



