

## TUTORIAL 5

$$\text{Q1) } |\psi\rangle = -3i|1\rangle + (2+i)|2\rangle + 4|3\rangle$$

$$|\varphi\rangle = 2|1\rangle - i|2\rangle + (2-3i)|3\rangle$$

$$\text{a) } \langle\varphi| = 2\langle 1| + i\langle 2| + (2+3i)\langle 3|$$

$$\begin{aligned}\text{b) } \langle\varphi|\psi\rangle &= (2\langle 1| + i\langle 2| + (2+3i)\langle 3|)(-3i|1\rangle + (2+i)|2\rangle + 4|3\rangle) \\ &= -6i\langle 1|1\rangle + i(2+i)\langle 2|2\rangle + 4(2+3i)\langle 3|3\rangle \\ &= -6i + 2i - 1 + 8 + 12i \\ \langle\varphi|\psi\rangle &= 7 + 8i\end{aligned}$$

$$\text{Q2) } |\psi\rangle = 3i|\varphi_1\rangle - 7i|\varphi_2\rangle$$

$$|\chi\rangle = -|\varphi_1\rangle + 2i|\varphi_2\rangle$$

$$\langle\varphi_i|\varphi_j\rangle = \delta_{ij}$$

$$\begin{aligned}\text{a) } |\psi+\chi\rangle &= |\psi\rangle + |\chi\rangle = (3i-1)|\varphi_1\rangle - 5i|\varphi_2\rangle \\ \langle\psi+\chi| &= (-3i-1)\langle\varphi_1| + 5i\langle\varphi_2|\end{aligned}$$

$$\begin{aligned}\text{b) } \langle\psi|\chi\rangle &= (-3i\langle\varphi_1| + 7i\langle\varphi_2|)(-|\varphi_1\rangle + 2i|\varphi_2\rangle) \\ &= +3i\langle\varphi_1|\varphi_1\rangle + 6\langle\varphi_1|\varphi_2\rangle - 7i\langle\varphi_2|\varphi_1\rangle - 14\langle\varphi_2|\varphi_2\rangle \\ &= 3i - 14\end{aligned}$$

$$\begin{aligned}\langle\chi|\psi\rangle &= (-\langle\varphi_1| - 2i\langle\varphi_2|)(3i|\varphi_1\rangle - 7i|\varphi_2\rangle) \\ &= -3i\langle\varphi_1|\varphi_1\rangle + 6\langle\varphi_2|\varphi_1\rangle + 7i\langle\varphi_1|\varphi_2\rangle - 14\langle\varphi_2|\varphi_2\rangle \\ &= -3i - 14\end{aligned}$$



