Modern Physics

Tutorial 13
Rotation Matrices and Bloch Sphere
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1 Rotation Operators

Consider the following operators:

$$\hat{R}_x(\theta) = e^{-i\theta\hat{\sigma_x}}$$
$$\hat{R}_z(\theta) = e^{-i\theta\hat{\sigma_z}}$$
$$\hat{R}_y(\theta) = e^{-i\theta\hat{\sigma_y}}$$

a) Apply these operators (one at a time) on a state $|\phi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$ and convince yourself that they rotate the state by angle θ along x, z, y axis respectively.

b) Write in words the action of the following operator on an arbitrary state

$$\begin{aligned}\hat{R}_n(\theta) &= e^{-i\theta\vec{n}\cdot\vec{\sigma}} \\ &= e^{-i\theta(n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)}\end{aligned}$$

c) Write the matrix for the following two operators:

$$\hat{A} = e^{-i\theta(n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)}$$
$$\hat{B} = e^{-i\theta n_x\hat{\sigma_x}} e^{-i\theta n_y\hat{\sigma_y}} e^{-i\theta n_z\hat{\sigma_y}}$$

d) Are the two matrices in part (c) equal? Are they? Or are they not? Are you sure you have not made a mistake?

2 Bloch Sphere

Consider the state $|\psi\rangle = cos(\frac{\theta}{2})|1\rangle + i\ sin(\frac{\theta}{2})|2\rangle$ of a two level system (such as a spin-$\frac{1}{2}$ particle).

a) Show that $|\psi\rangle$ is an eigenstate of $\hat{Q} = cos(\theta)\hat{\sigma_z} + sin(\theta)\hat{\sigma_y}$ and give the corresponding eigenvalue.

b) Give a pictorial representation of $|\psi\rangle$.

c) What operator transforms the state $|\psi\rangle$ into the state $|\phi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$? erive the result geometrically and verify it mathematically.
(Note: There is no unique answer to part (c))