Modern Physics

Tutorial 9
Quantum Mechanics
LUMS School of Science and Engineering

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1 Geometry and Angular Momentum

Find the angle between the angular momentum $l = 4$ and the z-axis for all posible orientations.

2 Spherical Harmonic Functions in Different Coordinate Systems

Consider a particle whose wave function is:

$$\psi(x, y, z) = \frac{1}{4\sqrt{\pi}} \frac{2z^2 - x^2 - y^2}{r^2} + \sqrt{\frac{3}{\pi}} \frac{xz}{r^2}$$

a) Calculate $\hat{\vec{L}}^2\psi(x, y, z)$ and $\hat{L}_z\psi(x, y, z)$. Find the total angular momentum of this particle.

b) If a measurement of the z-component of the orbital angular momentum is carried out, find the probabilities corresponding to finding the results $0, \hbar$, and $-\hbar$.

c) What is the probability of finding the particle at the position $\theta = \pi/3$ and $\phi = \pi/2$ within $d\theta = 0.03$ rad and $d\phi = 0.03$ rad.

Recall that:

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}}\frac{(3z^2 - r^2)}{r^2}$$
$$Y_{2,\pm 1} = \mp\sqrt{\frac{15}{8\pi}}\frac{(x \pm iy)z}{r^2}$$

3 Expectation Values

Find the expectation value of $\hat{P}^2$ and that of $\hat{P}^4$ in the ground state of the Hydrogen atom.

4 The Hydrogen Atom

Consider a hydrogen atom whose state at time $t = 0$ is given by:

$$\Psi(\vec{r}, 0) = \frac{1}{\sqrt{2}}\phi_{300}(\vec{r}) + \frac{1}{\sqrt{3}}\phi_{311}(\vec{r}) + \frac{1}{\sqrt{6}}\phi_{322}(\vec{r})$$

a) What is the time dependent wave function?

b) If the measurement of the energy were carried out, what values coud be found and with what probabilities?

c) Repeat part (b) for $\hat{L}^2$ and $\hat{L}_z$. That is, if the measurement of $\hat{L}^2$ and $\hat{L}_z$ were carried out, what values could be found and with what probabilities?