Modern Physics

Tutorial 8
Quantum Mechanics
LUMS School of Science and Engineering

November 5, 2012

1 Time Evolution of Two-State System

Let $|1\rangle$ and $|2\rangle$ be the eigenstates of a Hermitian operator $\hat{A}$ with eigenvalues λ_1 and λ_2 respectively. The hamiltonian operator is given by

$$\hat{H} = \delta(|1\rangle\langle 2| + |2\rangle\langle 1|), \tag{1}$$

where δ is just a real number with the dimensions of energy.

a) Clearly, $|1\rangle$ and $|2\rangle$ are not eigenstates of the Hamiltonian. Write down the normalized eigenstates of the Hamiltonian. What are their energy eigenvalues?

b) Suppose the system is known to be in state $|1\rangle$ at $t = 0$. Write down the state for $t > 0$.

c) What is the probability for finding th system in $|2\rangle$ for $t > 0$ if the system is known to be in state $|1\rangle$ at $t = 0$.

d) Repeat parts (b) and (c) if the initail state is $|\psi\rangle = \alpha|1\rangle + \beta|2\rangle$

2 Useful Identity

Prove the following identity:

$$\frac{d}{dt} < \hat{O} > = < \frac{d}{dt}\hat{O} > + \frac{1}{i\hbar} < [hatO, hatH] >$$

3 Particle in a Box

Consider the infinite well between $x = 0$ and $x = a$. What is the state of the particle at $t > 0$ if the state at $t = 0$ is given by:

$$\Psi(x, 0) = Ax(a - x).$$