Modern Physics

Tutorial 7
Quantum Mechanics
LUMS School of Science and Engineering

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1 Operators and Hermitian Conjugates

Consider the two states $|\psi\rangle = i|\phi_1\rangle + 3i|\phi_2\rangle - |\phi_3\rangle$ and $|\chi\rangle = |\phi_1\rangle + -i|\phi_2\rangle + 5i|\phi_3\rangle$, where $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$ are orthonormal.

a) calculate $\langle\psi|\psi\rangle$, $\langle\chi|\chi\rangle$, $\langle\psi|\chi\rangle$, $\langle\chi|\psi\rangle$, and infer $\langle\psi+\chi|\psi+\chi\rangle$.

b) calculate $|\psi\rangle\langle\chi|$ and $|\chi\rangle\langle\psi|$. Are they equal?

c) Find the hermitian conjugates of $|\psi\rangle$, $|\chi\rangle$,$|\psi\rangle\langle\chi|$ and $|\chi\rangle\langle\psi|$.

2 Orthgonal Kets

Consider two states $|\psi_1\rangle = |\phi_1\rangle + 4i|\phi_2\rangle + 5|\phi_3\rangle$ and $|\psi_2\rangle = b|\phi_1\rangle + 4|\phi_2\rangle - 3i|\phi_3\rangle$, where $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ are othonormal kets, and where b is a constant.

Find the value of b so that $|\psi_1\rangle$ and $|\psi_1\rangle$ are orthogonal.

3 Ket Algebra

If $|\psi\rangle = |\phi_1\rangle + |\phi_2\rangle$ and $|\chi\rangle = |\phi_1\rangle$ - $|\phi_2\rangle$, prove the following relations (note that $|\phi_1\rangle$ and $|\phi_2\rangle$ are not orthonormal):

a) $\langle\psi|\psi\rangle + \langle\chi|\chi\rangle = 2\langle\phi_1|\phi_1\rangle + 2\langle\phi_2|\phi_2\rangle$,

b) $\langle\psi|\psi\rangle - \langle\chi|\chi\rangle = 2\langle\phi_1|\phi_2\rangle + 2\langle\phi_2|\phi_1\rangle$.

4 Normalization and Expectation Value

Consider a state which is given in terms of three orthonormal vectors $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ as follows:

$$|\psi\rangle = \sqrt{\frac{1}{15}}|\phi_1\rangle + \sqrt{\frac{1}{3}}|\phi_2\rangle + \sqrt{\frac{1}{5}}|\phi_3\rangle$$

where $|\phi_n\rangle$ are eigenstates of an operator $\hat{B}$ such that $\hat{B}|\phi_n\rangle = (3n^2 - 1)|\phi_n\rangle$ with n= $1, 2, 3$.

a) Find the norm of the state $|\psi\rangle$.

b) Normalize $|\psi\rangle$.

c) Find the expectation value of $\hat{B}$ for the state $|\psi\rangle$.

d) Find the expectation value of $\hat{B}^2$ for the state $|\psi\rangle$.

5 Finding the Eigenvalue and the Eigenstate

A two-state system is characterized by the Hamiltonian:

$$\hat{H} = H_{11}|1\rangle\langle 1| + H_{22}|2\rangle\langle 2| + H_{12}[|1\rangle\langle 2| + |2\rangle\langle 1|] \tag{1}$$

where H_{11}, H_{12}, and H_{22} are real number with the dimension of energy, and $|1\rangle$ and $|2\rangle$ are orthonormal.

Find the energy eigenvalues and the corresponding eigenvectors. Make sure that your answer makes good sense for $H_{12} = 0$.

6 Another Example of Eigenvalue and Eigenvectors Problem

Consider a system whose Hamiltonian is given by $\hat{H} = \alpha(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|)$, where α is a real number having the dimensions of energy and $|\phi_1\rangle, |\phi_2\rangle$ are normalized eigenstates of a Hermitian operator $\hat{A}$ that has no degenerate eigenvalues.

a) Show that $|\phi_1\rangle$ and $|\phi_2\rangle$ are not eigenstates of $\hat{H}$.

b) Calculate the commutators $[\hat{H}, |\phi_1\rangle\langle\phi_1|]$ and $[\hat{H}, |\phi_2\rangle\langle\phi_2|]$ then find the relation that may exist between them.

c) Find the normalized eigenstates of $\hat{H}$ = and their correspondng energy eigenvalues.

7 Wavefunctions for a Particle

At time t= 0 a particle is represented by the wave function

$$\Psi(x,0) = \begin{cases} A\frac{x}{a} & \text{if } 0 \le x \le a, \\ A\frac{b-x}{b-a} & \text{if } a \le x \le b, \\ 0 & \textit{otherwise} \end{cases}$$

where A, a, and b are constants.

a) Normalize Ψ (that is, find A, in terms of a and b).

b) Sketch $\Psi(x, 0)$ as a function of x.

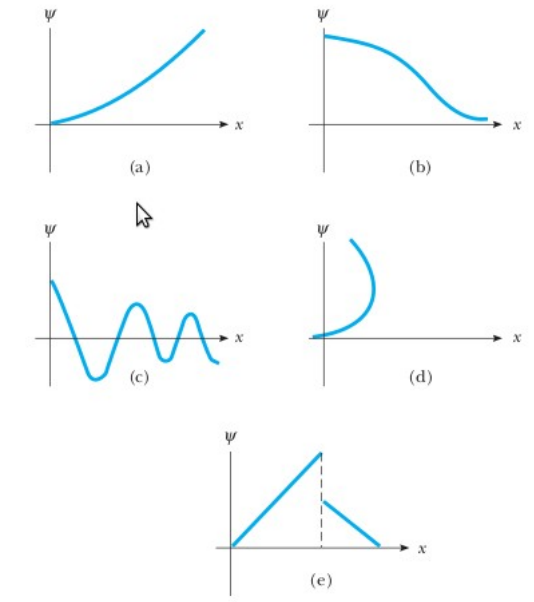

Figure 1: Candidates for the Wavefunctions

c) Where is the particle most likely to be found, at t= 0.

d) What is the probability of finding the particle to the left of a? Check your answer in the limiting case b = a and b = 2a.

e) What is the expectation value of x?

f) What is the expectation value of x^2?

8 Legitimate Wavefunctions

Of the functions graphed in figure1, which are candidates for the Schrodinger wavefunction of an actual physical system? For those that are not, state why they fail to qualify.