

# Modern Physics

## Tutorial 6

### Quantum Mechanics

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## 1 Operators and Hermitian Conjugates

Consider the states  $|\psi\rangle = 9i|\phi_1\rangle + 2|\phi_2\rangle$  and  $|\chi\rangle = -\frac{i}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle$ , where  $|\phi_1\rangle$  and  $|\phi_2\rangle$  are orthonormal.

- calculate the operators  $|\psi\rangle\langle\chi|$  and  $|\chi\rangle\langle\psi|$ . Are they equal?
- Find the hermitian conjugates of  $|\psi\rangle$ ,  $|\chi\rangle$ ,  $|\psi\rangle\langle\chi|$  and  $|\chi\rangle\langle\psi|$

## 2 Expectation Value

Consider the state

$$|\psi\rangle = \sqrt{\frac{1}{2}}|\phi_1\rangle - \sqrt{\frac{2}{5}}|\phi_2\rangle + \sqrt{\frac{1}{10}}|\phi_3\rangle$$

which is given in terms of three orthonormal eigenstates  $|\phi_1\rangle$ ,  $|\phi_2\rangle$ , and  $|\phi_3\rangle$  of an operator  $\hat{B}$  such that  $\hat{B}|\phi_n\rangle = n^2|\phi_n\rangle$ .

Find the expectation value of  $\hat{B}$  for the state  $|\psi\rangle$ .

## 3 Common Eigenvalues!!!

Suppose  $|i\rangle$  and  $|j\rangle$  are eigenkets of some Hermitian operator  $\hat{A}$ . Under what conditions can we say that  $|i\rangle + |j\rangle$  is also an eigenket of  $\hat{A}$ ? Justify your answer.

## 4 Finding the Eigenvalue and the Eigenstate

The Hamiltonian operator for a two state system is given by

$$\hat{H} = a(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|) \quad (1)$$

where  $a$  is a number with the dimension of energy.

Find the energy eigenvalues and the corresponding eigenvectors (as a linear combination of  $|1\rangle$  and  $|2\rangle$ ).