

Modern Physics

Tutorial 5
Quantum Mechanics
LUMS School of Science and Engineering

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1 Inner Product

Consider the two kets:

$$\begin{aligned} |\psi\rangle &= (-3i)|1\rangle + (2+i)|2\rangle + 4|3\rangle \\ |\phi\rangle &= 2|1\rangle - i|2\rangle + (2-3i)|3\rangle \end{aligned}$$

- Find the bra $\langle\phi|$.
- Evaluate the inner product $\langle\phi|\psi\rangle$.

2 Bra-Ket Algebra

Consider the states

$$\begin{aligned} |\psi\rangle &= 3i|\phi_1\rangle - 7i|\phi_2\rangle \\ |\chi\rangle &= -|\phi_1\rangle + 2i|\phi_2\rangle, \end{aligned}$$

where $|\phi_1\rangle$ and $|\phi_2\rangle$ are orthonormal.

- Calculate $|\psi + \chi\rangle$ and $\langle\psi + \chi|$
- calculate the inner products $\langle\psi|\chi\rangle$ and $\langle\chi|\psi\rangle$
- verify the following inequality:

$$|\langle\psi|\chi\rangle|^2 \leq \langle\psi|\psi\rangle\langle\chi|\chi\rangle$$

- verify the following inequality:

$$\sqrt{\langle\psi + \chi|\psi + \chi\rangle} \leq \sqrt{\langle\psi|\psi\rangle} + \sqrt{\langle\chi|\chi\rangle}$$

3 Normalization and Orthogonal Kets

Consider the two states

$$\begin{aligned}|\psi_1\rangle &= 2i|\phi_1\rangle + |\phi_2\rangle - a|\phi_3\rangle + 4|\phi_4\rangle \\ |\psi_2\rangle &= 3|\phi_1\rangle - i|\phi_2\rangle + 5|\phi_3\rangle - |\phi_4\rangle,\end{aligned}$$

where $|\phi_1\rangle$, $|\phi_2\rangle$, $|\phi_3\rangle$, and $|\phi_4\rangle$ are orthonormal, and where a is a constant. Find the value of a so that $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthogonal.