Modern Physics

Week 5 - Tutorial

28^{th} September, 2012

Problem 1. Show that a photon cannot transfer all of its energy to a free electron. This effect is not seen in the photoelectric effect. Argue why?

Problem 2. Find the uncertainty in the location of a particle, in terms of its de Broglie wavelength λ, so that the uncertainty in its velocity is equal to its velocity.

Problem 3. A harmonic oscillator has energy,

$$E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2,$$

where ω is the frequency of the oscillator. Classically x = 0, p = 0 results in a minimum energy equal to zero. Now use the uncertainty principle to estimate the minimum energy. Assume $\Delta p \approx p$ and $\Delta x \approx x$.

Problem 4. In an interference experiment, the arrival of the electrons on the screen is said to be "random". The tossing of a coin in real life is also random. Are the two behaviors - electrons in an interference experiment and tossing coins - similar or different and why?

Problem 5. A student A performs an interference experiment on a particular day and observes a pattern of fringes. She then repeats the same experiment some other day. Will the two patterns be different or identical? Assuming ideal environmental conditions, is quantum certainty "reproducible"?

Problem 6. Special relativity tells us that no material particle can travel faster than light. Consider an electron with relativistic energy $E = \gamma mc^2, m$ being the relativistic mass. The energy can also be expressed in terms of the relativistic momentum p and the rest mass, $E^2 = (mc^2)^2 + (pc)^2$. Find the phase and group velocities of the electron. You will notice that under some conditions, the phase velocity can exceed the speed of light. How do you resolve this paradox?

Problem 7. Consider two kets, $|\psi\rangle = i|\phi_1\rangle + 3i|\phi_2\rangle - |\phi_3\rangle$ and $|\chi\rangle = |\phi_1\rangle - i|\phi_2\rangle + 3i|\phi_3\rangle$, where $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ are orthonormal.
i). Calculate $\langle\psi|\psi\rangle, \langle\psi|\chi\rangle, \langle\chi|\psi\rangle$ and $\langle\chi|\chi\rangle$.
ii). Write $|\psi\rangle$ and $|\chi\rangle$ in the normalized form.
iii). Add $|\psi\rangle$ and $|\chi\rangle$ and then find $\langle\psi+\chi|\psi+\chi\rangle$.