Modern Physics

Final Exam - Review Session 1
Pre-Mid Term Syllabus
LUMS School of Science and Engineering

December 11, 2012

1 Decay of the Muon

The muon is an unstable particle that spontaneously decays into an electron and two neutrinos. If the number of muons at $t = 0$ is N_o, the number at time t is given by $N = N_o e^{-\frac{t}{\tau}}$, where τ is the mean lifetime, equal to 2.2μs. Suppose the muons move at a speed of 0.95c and there are 5.0x10^4 muons at $t = 0$. (a) What is the observed lifetime of the muons? (b) How many muons remain ater travelling a distance of 3.0km?

2 Lorentz Transformation

An observer in frame S sees lightning simultaneously strike two points 100m apart. The first strike occurs at $x_1 = y_1 = z_1 = t_1 = 0$ and the second at $x_2 = 100m$, $y_2 = z_2 = t_2 = 0$. (a) What are the coordinates of these two events in a frame $S^{'}$ moving in the standard configuration at 0.7c relative to S? (b) How far apart are the events in $S^{'}$? (c) Are the two events simultaneous in $S^{'}$? If not, what is the difference in time between the events, and which event occurs first?

3 Addition of Velocities

A spaceship moves away from Earth at a speed v and fires a shuttle craft in the forward direction at a speed v relative to the ship. The pilot of the shuttle craft launches a probe at speed v relative to the shuttle craft. Determine (a) the speed of the shuttle craft relative to Earth, and (b) the speed of the probe relative to Earth.

4 Electron-Positron Annihilation

An electron having kinetic energy $K = 1.000\text{M}eV$ makes a head-on collision with a positron at rest. In the collision the two particles annihilate each other

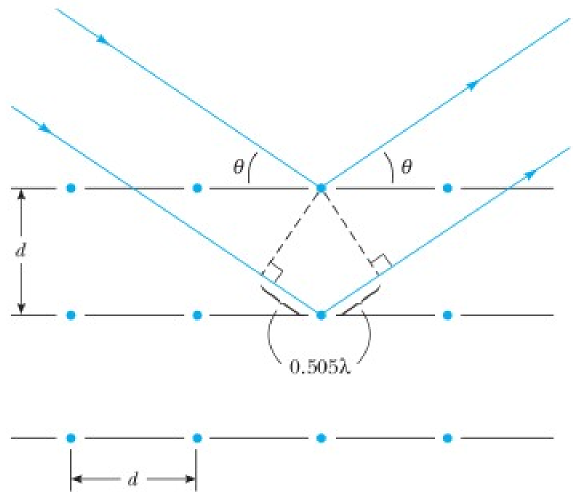

Figure 1: figure for question 7

and are replaced by two γ rays of equal energy, each traveling at equal angles ϕ with the electron's direction of motion. Find the energy E, momentum p, and angle of emission ϕ of the γ rays.

5 Relativistic Collisions

A particle of mass m moving along the x-axis with a velocity component $+u$ collides head-on and sticks to a particle of mass $m/3$ moving along the x-axis with the velocity component $-u$. What is the mass M of the resulting particle?

6 Photoelectric Effect

Photons of wavelength $450nm$ are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius $20cm$ by a magnetic field whose strength is 2.0x10^{-5}T. What is the work function of the metal?

7 The Davisson-Germer Experiment

Figure below shows the top three planes of a crystal with planar spacing d. If $2d$ $sin\theta = 1.01\lambda$ for the two waves shown, and high energy electrons of wavelength λ penetrate many planes deep into the crystal, which atomic plane produces a wave that cancels the surface reflection?

8 Uncertainty Principle

A beam of electrons is to be fired over a distance of 10^4 km. If the size of the initial packet is $10^{-3}m$, what will be its size upon arrival, if its kinetic energy is (a) $13.6eV$; (b) 100 MeV?

9 Dirac Notation

Consider a state which is given in terms of three orthonormal vectors $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ as follows:

$$|psi\rangle = \frac{1}{\sqrt{15}}|\phi_1\rangle + \frac{1}{\sqrt{3}}|\phi_2\rangle + \frac{1}{\sqrt{5}}|\phi_3\rangle$$

where $|\phi_n\rangle$ are eigenstate to an operator $\hat{B}$ such that: $\hat{B}|\phi_n\rangle = (3n^2 - 1)|\phi_n\rangle$ with n = 1,2,3.

(a) Find the norm of the state $|\psi\rangle$.

(b) Find the expectation value of $\hat{B}$ for the state $|\psi\rangle$.

(b) Find the expectation value of $\hat{B}^2$ for the state $|\psi\rangle$.

10 Orthonormality Condition

Find the constant c so that the states $|\psi\rangle = c|\psi_1\rangle + 5|\phi_2\rangle$ and $|\chi\rangle = 3c|\psi_1\rangle - 4|\phi_2\rangle$ are orthogonal; consider $|\phi_1\rangle$ and $|\phi_2\rangle$ to be orthonormal.

11 Particle in a Box

A particle in an infinite potential box with walls at $x = 0$ and $x = a$ has the following wavefunctions at some initial time:

$$\psi(x) = \frac{1}{\sqrt{5a}} sin(\frac{\pi x}{a}) + \frac{2}{\sqrt{5a}} sin(\frac{3\pi x}{a})$$

(a) Find the possible results of the measurement of the system's energy and the corresponding probabilities.

(b) Find the form of the wavefunction after such a measurement.

(c) If the energy is measured again immediately afterwards, what are the relative probabilities of th epossible outcomes?

12 Compatibility of Energy and Momentum

Show that the momentum and the total energy can be measured simultaneously only when the potential is constant everywhere.

13 Transitions in an Infinite Well

An electron is contained in a one dimensional box of width $0.100nm$. (a) Draw an energy-level diagram for the electron for levels up to $n = 4$. (b) Find the wavelengths of *all* photons that can be emitted by the electron in making transitions that would eventually get it from the $n = 4$ state to the $n = 1$ state.

14 Symmetric Infinite Well

Consider a particle moving in a one-dimensional box with walls at $x = -L/2$ and $x = L/2$.

(a) Write down the wavefunctions and probability densities for the state $n = 1$, $n = 2$, and $n = 3$.

(b) Sketch the wavefunctions and probability densities.

15 Nonstationary States

Consider a particle in an infinite square well described initially by a wave that is a superposition of the ground and the first excited states of the well:

$$\Psi(x, 0) = C[\psi_1(x) + \psi_2(x)]$$

(a) Show that the value $C = 1/\sqrt{2}$ normalize this wave, assuming ψ_1 and ψ_2 are themselves normalized.

(b) Show that the superposition is *not* a stationary state, but the average energy in this state is the arithmetic mean $(E_1 + E_2)/2$ of the ground- and first excited-state energies E_1 and E_2.