Modern Physics

Review Session 4
Quantum Mechanics
LUMS School of Science and Engineering

November 14, 2012

1 Particle in a Well

Consider a one-dimensional particle which is confined within the region $0 \leq x \leq a$ and whose wavefunction is $\Psi(x,t) = sin(\pi x/a)exp(-i\omega t)$.

a) Find the potential V(x).

b) Calculate the probability of finding the particle in the interval $a/4 \leq x \leq 3a/4$.

2 1-D SHO

Consider the particle of mass m oscillating with the angular frequency ω. The state of the particle at time $t = 0$ is given by

$$|\psi\rangle = \sqrt{\frac{1}{2}}|2\rangle + \sqrt{\frac{1}{4}}|3\rangle + \sqrt{\frac{1}{4}}|6\rangle$$

a) If the energy of the particle is measured, what values will be obtained and with what probability?

b) What is the state of the particle at any later time t?

3 Uncertainty in Position

Calculate the width of the probability density distribution for r for the hydrogen atom in its ground state.

4 Hydrogen Atom

A hydrogen atom is in the $6d$ state.

a) What is the principal quantum number, n?

b) What is the energy of the atom?

c) What are the values for the orbital quantum number and the magnitude of the electron's orbital angular momentum?

d) What are the possible values for the magnetic quantum number? For each value, find the corresponding z-component of the electron's orbital quantum number and the angle that the orbital angular momentum vector makes with the z-axis?

5 Hydrogen Atom Again

Consider a hydrogen atom whose state at time $t = 0$ is given by

$$\Psi(\vec{r}, 0) = \frac{1}{\sqrt{2}}\phi_{300}(\vec{r}) + \frac{1}{\sqrt{3}}\phi_{311}(\vec{r}) + \frac{1}{\sqrt{2}}\phi_{322}(\vec{r})$$

a) What is the time-dependent wave function?

b) If a measurement of the energy were carried out, what values could be found and with what probabilities?

c) Repeat part (b) for $\hat{L}^2$ and $\hat{L}_z$. That is, if a measurement of $\hat{L}^2$ and $\hat{L}_z$ were carried out, what values could be found and with what probabilities?

6 Hydrogen Atom Once Again

Consider a hydrogen atom whose state at time $t = 0$ is given by

$$\Psi(\vec{r}, 0) = A\phi_{200}(\vec{r}) + \frac{1}{\sqrt{5}}\phi_{311}(\vec{r}) + \frac{1}{\sqrt{3}}\phi_{422}(\vec{r}),$$

where A is a normalization constant.

a) Find A so that the state is normalized.

b) Find the state of this atom at any later time t.

c) If a measurement of the energy were carried out, what values would be found and with what probabilities?

d) Find the mean energy of the atom.

7 Degenerate Eigenvalues in QM

Prove that the nth energy level of an atom has degeneracy equal to n^2.

8 Spherical Harmonics in Cartesian Coordinates

Consider the function $\psi(\vec{r}) = -A(x+iy)e^{-r/2a_o}$, where a_o is the Bohr radius and A is a real constant.

a) Is $\psi(\vec{r})$ and eigenfunction to $\hat{\vec{L}}^2$ and $\hat{L}_z$? If yes, write $\psi(\vec{r})$ in terms of $R_{nl}(r)Y_{lm}(\theta,\phi)$ and find the values of the quantum numbers n, m, l.

b) FInd the constant A so that $\psi(\vec{r})$ is normalized.

c) Find the mean value of r and the most probable value of r in this state.

9 Electrons inside Arctic Polar Coordinates

Calculate the probability that the electron in a hydrogen atom would be found within 30^o of the $x-y$ plane, irrespective of radius, for,

a) $l = 0$, $m_l = 0$,

b) $l = 1$, $m_l = +1$,

c) $l = 2$, $m_l = +2$.

10 Hydrogen-like Atom

a) Determine the qunatum numbers l and m_l for the He^+ ion in the state corresponding to $n = 3$. b) What is the energy of this state?