Modern Physics

Review Session
Quantum Mechanics
LUMS School of Science and Engineering

October 16, 2012

1 Dirac Notation's Basics

In the following expressions, where $\hat{A}$ is an operator, specify the nature of each expressions (i.e, specify whether it is an operator, a bra or a ket) then find its hermitian conjugate.

a) $\langle\phi|\hat{A}|\psi\rangle\langle\psi|$
b) $\hat{A}|\phi\rangle\langle\psi|$
c) $\langle\phi|\hat{A}|\psi\rangle|\phi\rangle\hat{A}$
d) $\langle\phi|\hat{A}|\psi\rangle|\psi\rangle + i\hat{A}|\psi\rangle$
e) $|\phi\rangle\langle\phi|\hat{A} - i\hat{A}|\psi\rangle\langle\psi|$

2 Fun with Commutators

Consider an operator $\hat{A}$ so that $[\hat{A}, \hat{A}^\dagger] = 1$.

a) Evaluate the commutators $[\hat{A}^\dagger\hat{A}, \hat{A}]$ and$[\hat{A}^\dagger\hat{A}, \hat{A}^\dagger]$.

b) If the actions of $\hat{A}$ and $\hat{A}^\dagger$ on the states are given by $\hat{A}|a\rangle = \sqrt{a}|a-1\rangle$ and $\hat{A}^\dagger|a\rangle = \sqrt{a+1}|a+1\rangle$ and the states $|a\rangle$ and $|a'\rangle$ are orthonormal, calculate $\langle a|\hat{A}|a+1\rangle$, $\langle a+1|\hat{A}^\dagger|a\rangle$, $\langle a|\hat{A}^\dagger\hat{A}|a\rangle$, $\langle a|\hat{A}\hat{A}^\dagger|a\rangle$.

c) Calculate $\langle a|(\hat{A}+\hat{A}^\dagger)^2|a\rangle$ and $\langle a|(\hat{A}-\hat{A}^\dagger)^2|a\rangle$.

3 Differential Operators and the Eigenvalues

Consider a one dimensional particle which moves along the xaxis and whose Hamiltonain is given by:

$$\hat{H} = -\alpha \frac{d^2}{d^2x} + 16\alpha \hat{X}^2$$

where α is a real constant having the dimensions of energy.

a) Is $\psi(x) = Ae^{-2x^2}$ an eigenfunction of $\hat{H}$. If yes, find the enegy eigenvalues.

b) Calculate the probability of finding the particle anywhere along the negative x-axis.

c) Find the energy eigenvalues corresponding to the wavefunction $\phi(x) = 2x\psi(x)$.

d) Are $\phi(x)$ and $\psi(x)$ orthogonal?

4 Particle in a Box

Consider a particle in an infinite square well whose wavefunction is given by

$$\Psi(x) = \begin{cases} Ax(a^2 - x^2) & \text{if } 0 \leq x \leq a, \\ 0 & elsewhere \end{cases}$$

where A is a real constant.

a) Find A so that $\Psi(x)$ is normalized.

b) Calculate the probability of finding $n^2\pi^2\hbar^2/(2ma^2)$ for a measurement of energy.

c) Calculate the position and momentum uncertainties, Δx and Δp, and the product $\Delta x \Delta p$.

5 Wavefunction in the Presence of Forces

In a region of space, a particle with zero energy has a wavefunction:

$$\psi(x) = Axe^{-\frac{x^2}{L^2}}$$

a) Find the potential energy U as a function of x.

b) Make a sketch of U(x) versus x.

6 Harmonic Oscillator in the Classical Limit

Estimate the quantum number n for the particle oscillating with the energy E = $1J$.

Then explain why we can not observe the discrete energy levels of the pendulum in the physics lab.

7 The Quantum Oscillator in the Nonclassical Region

Consider the harmonic oscillator in its ground state.

a) Find the classical turning points.

b) Calculate the probability the a quantum oscillator in its ground state will be found in the classical forbidden region.