

Modern Physics

Final Exam - Review Session 3
Post-Mid Term Syllabus
LUMS School of Science and Engineering

December 13, 2012

1 Two Level System

Consider a two level system with $E_1 < E_2$. There is a time-dependent potential that connects the two levels as follows:

$$\begin{aligned}V_{11} &= V_{22} = 0, \\V_{12} &= \gamma e^{i\omega t}, \\V_{21} &= \gamma e^{-i\omega t}\end{aligned}$$

At $t = 0$, it is known that only the lower level is populated - that is, $c_1(0) = 1$ and $c_2(0) = 0$.

- Find $|c_1(t)|^2$ and $|c_2(t)|^2$ exactly by solving the coupled differential equations.
- Do the same with the approximation that γ is very very small.

2 Free Fermi Gas Model

The density of Copper is 8.96 gm/cm^3 , and its atomic weight is 63.5 gm/mole .

- Calculate the fermi energy for copper.
- Calculate the fermi momentum, fermi velocity, and the fermi wave number.
- At what temperature would the characteristics thermal energy ($k_B T$, where k_B is the Boltzmann constant and T is the temperature in kelvin) equal the fermi energy, for copper?
- Calculate the Pressure of copper, in the electron gas model.

3 Quantum Logic Gates

This question goes through a particular example of the statement that any operation can be constructed from a combination of a control gate and arbitrary

rotations of the individual qubits. For trapped ions the most straightforward logic gate to build is a controlled 'rotation' of Qubit 2 when Qubit 1 is $|1\rangle$, i.e.

$$\hat{U}_{CROT} = |00\rangle\langle 00| + |10\rangle\langle 10| + |01\rangle\langle 01| - |11\rangle\langle 11|$$

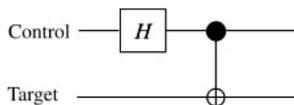
- (a) Write \hat{U}_{CROT} as a 4x4 matrix and show that it is unitary.
- (b) Write the Hadamard transformation for Qubit 2, $\hat{U}_H(2)$ as a 4x4 matrix with the same basis states as in (a), and show that it is unitary.
- (c) Find the combination of \hat{U}_{CROT} , $\hat{U}_H(2)$ and $\hat{U}_H^\dagger(2)$ that gives the CNOT gate.

4 Quantum Gate Circuit

Calculate the output of the quantum gate circuit shown in fig. 1 when the input wavefunction is

- (a) $|00\rangle$,
- (b) $|01\rangle$,
- (c) $|10\rangle$,
- (d) $|11\rangle$.

Assume that it is the first qubit that undergoes the Hadamard operation.



5 Bohat Sarey Hadamard Gates

Verify the following identities:

$$\begin{aligned} & (H \otimes H \otimes \dots \otimes H)|00 \dots 0\rangle \\ &= \frac{1}{\sqrt{2^n}} ((|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \\ & \quad \otimes \dots \otimes (|0\rangle + |1\rangle)) \\ &= \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle. \end{aligned}$$