Modern Physics

Final Exam - Review Session 2
Post-Mid Term Syllabus
LUMS School of Science and Engineering

December 12, 2012

1 Time Evolution 1

The initial state of a sysytem is given in terms of four orthonormal energy eigenfunctions $|\phi_1\rangle$, $|\phi_2\rangle$, $|\phi_3\rangle$, and $|\phi_4\rangle$ as follows:

$$|\psi_0\rangle = \frac{1}{\sqrt{3}}|\phi_1\rangle + \frac{1}{2}|\phi_2\rangle + \frac{1}{\sqrt{6}}|\phi_3\rangle + \frac{1}{2}|\phi_4\rangle$$

(a) If the four kets $|\phi_1\rangle$, $|\phi_2\rangle$, $|\phi_3\rangle$, and $|\phi_4\rangle$ are eigenvectors to the Hamiltonian $\hat{H}$ with the energies E_1, E_2, E_3, E_4 respectively, find the state at any later time t.

(b) What are the possible results of measuring the energy and with what probability will they occur?

(c) Find the expectation value of the sysytem's Hailtonian at $t = 0$ and $t = 10s$.

2 Time Evolution 2

Consider a sysytem whose wavefunction at time $t = 0$ is given by

$$\psi(x, 0) = \frac{5}{\sqrt{50}}\phi_0(x) + \frac{4}{\sqrt{50}}\phi_1(x) + \frac{3}{\sqrt{50}}\phi_2(x)$$

where $\psi_n(x)$ is the wavefunction of the nth excited state for a harmonic oscillator of energy $E_n = \hbar\omega(n + \frac{1}{2})$.

(a) Find the average energy of this system.

(b) Find the state $\psi(x,t)$ at any later time t and the average value of the energy; compare this with the result of part (a).

(c) Find the expectation value of the operator $\hat{X}$ with respect to the state $\psi(x,t)$

3 Hydrogen Atom 1

Consider a Hydrogen atom which is in its ground state; the ground state wavefunction is given by

$$\Psi(r,\theta,\phi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

where a_0 is the Bohr radius.

(a) Find the most probable distance between the electron and the proton when the Hydrogen atom is in its ground state.

(b) Find the average distance between the electron and the proton.

4 Positronium

Positronium is a hydrogen-like atom consisting of a positron and an electron revolving around each other. Find the Bohr radius and the allowed energies of the system.

5 Hydrogen Atom 2

(a) Find $< r >$ and $< r^2 >$ for an electron in the ground state of hydrogen.

(b) Find $< x >$ and $< x^2 >$ for an electron in the ground state of hydrogen. *Hint* : This requires no new integration.

(c) Find $< x^2 >$ in the state $n = 2$, $l = 1$, $m = 1$.

6 Hydrogen like Ions

(a) Determine the quantum numbers l and m_l for the li^{2+} ion in the state for which $n = 1$ and $n = 2$.

(b) What are the enegies of these states?

7 Angular Momentum

Consider a system which is in the state

$$\psi(\theta, \phi) = \sqrt{\frac{2}{13}} Y_{3,-3} + \sqrt{\frac{3}{13}} Y_{3,-2} + \sqrt{\frac{3}{13}} Y_{3,0} + \sqrt{\frac{3}{13}} Y_{3,2} + \sqrt{\frac{2}{13}} Y_{3,3}$$

(a) If $\hat{L}_z$ is measured, what values will one obtain and with what probabilities.

(b) If after a measurement of $\hat{L}_z$ we find $l_z = 2\hbar$, calculate the uncertainties $\Delta\hat{L}_x$ and $\Delta\hat{L}_y$ and their product $\Delta\hat{L}_x\Delta\hat{L}_y$

8 Energy Levels of a Symmetric Rotator

(a) Calculate the energy eigenvalues of an axially symmetric rotator and find the degeneracy of each energy level. Note that the Hamiltonian of an axially symmetric rotator is given by

$$\hat{H} = \frac{\hat{L}_x^2 + \hat{L}_y^2}{2I_1} + \frac{\hat{L}_z^2}{2I_2}$$

(b) From part (a), infer the energy eigenvalues for the various levels of $l = 3$.

(c) In the case of a rigid rotator (i.e, $I_1 = I_2 = I$), find the energy expression and the corresponding degeneracy relation.

(d) Calculate the orbital quantum number l and the corresponding energy degeneracy for a rigid rotator where the magnitude of the total angular momentum is $sqrt56\hbar$