LUMS School of Science and Engineering

PHY-201 **Quiz # 1 - Solutions** September 22, 2012

Problem 1

In the rest frame of the earth, we can perceive all motion to be along the x axis. So lets say ship 1 has speed v_1 along the $+x$ direction, and ship 2 has speed v_2 along the $-x$ direction. Take the reference frame of the earth as S and the reference frame of ship 1 as S', both assumed in standard configuration, with velocity parameter v. Then clearly, $v = v_1$. Moreover, the speed of ship 2 in S is $u_{x2} = -v_2$, and that in S' can be labeled u'_{x2}. Invoking the usual velocity transformation formula gives:

$$u'_{x_2} = \frac{u_{x_2} - v}{1 - v u_{x_2}/c^2} = -\ \frac{v_1 + v_2}{1 + v_1 v_2/c^2}\ . \tag{1}$$

Thus as viewed from ship 1, ship 2 has the speed given above, and its lorentz contracted length can be calculated according to this speed. Noting that the rest length of ship 2 is $2L$ and using the length contraction formula we get:

$$L_2 = \frac{L_{0_2}}{\gamma_2} = 2L\sqrt{1 - \frac{(v_1 + v_2)^2}{(1 + v_1 v_2/c^2)^2 c^2}}\ . \tag{2}$$

Now it is straight forward to see that the time taken by ship 2 to fly by ship 1, is precisely its length divided by its speed (as viewed from ship 1). That is, it is the ratio of the quantity in (2) to the modulus of the quantity in (1):

$$\Delta t = 2L\sqrt{\frac{(1 + v_1 v_2/c^2)^2}{(v_1 + v_2)^2} - \frac{1}{c^2}}\ . \tag{3}$$

This is the time taken by ship 2 to fly by ship 1, as measured by its pilot.

Problem 2

(a) Here the emitted frequency in the rest frame of the earth is ν_0. In this frame, ship 1 recedes at speed v_1. Moreover there is perfect symmetry between the earth and the ship. Thus as viewed from the ship, the earth (source) recedes at speed v_1, emitting waves of proper frequency ν_0. And the ship receives frequency ν_1. In this familiar configuration, the Doppler's formula reads:

$$\nu_1 = \sqrt{\frac{c - v_1}{c + v_1}}\nu_0\ . \tag{4}$$

This provides one of the required answers. Moreover ship 1 reradiates with the same frequency, and a red shifted frequency is observed at ship 2. The red shift implies that ships 1 and 2 must be moving away from each other. So as viewed from ship 1, ship 2 moves away with speed v_2. And from symmetry, it must be that as viewed from ship 2 (the new observer), ship 1 (source emitting proper frequency ν_1) moves away at speed v_2. The Doppler's shift reads:

$$\nu_2 = \sqrt{\frac{c - v_2}{c + v_2}}\nu_1\ . \tag{5}$$

Finally, using (4) in (5) gives the required answer:

$$\nu_2 = \sqrt{\frac{c - v_2}{c + v_2}}\sqrt{\frac{c - v_1}{c + v_1}}\nu_0\ . \tag{6}$$

(b) Take the earth frame to be S and that of ship 1 to be S'. Let S and S' be in standard configuration with velocity parameter v. Then clearly: $v = v_1$. Moreover in the rest frame of ship 1, ship 2 has speed v_2 in the same direction. Thus $u'_{x_2} = v_2$. And the velocity transformation formula (the inverse) reads:

$$u_{x_2} = \frac{u'_{x_2} + v_1}{1 + v_1 u'_{x_2}/c^2} = \frac{v_1 + v_2}{1 + v_1 v_2/c^2} \ , \tag{7}$$

the speed of ship 2 relative to ship 1.

Problem 3

Here a stationary parent nucleus (rest mass M) undergoes decay, forming a daughter nucleus (rest mass m) and a photon. Clearly the initial momentum is zero (before decay). And momentum conservation requires that the daughter nucleus and the photon be emitted in opposite directions with equal momenta. The most obvious thing to do is to write out the equations of momentum and energy conservation. Denoting the speed of the daughter nucleus by v, the corresponding lorentz factor by γ, and the photon wavelength by λ, we have:

$$\gamma m v = \frac{h}{\lambda} \ , \tag{8}$$

$$M c^2 = \gamma m c^2 + \frac{hc}{\lambda} \ . \tag{9}$$

Eliminating h/λ between the above two equations allows us to write:

$$\gamma m (c + v) = M c \ . \tag{10}$$

Squaring and substituting γ in terms of v, allows us to solve for v:

$$v = \frac{M^2 - m^2}{M^2 + m^2} \ c \ . \tag{11}$$

Using this v, we can now compute γ:

$$\gamma = \frac{1}{\sqrt{1 - [\frac{M^2 - m^2}{M^2 + m^2}]^2}} \ . \tag{12}$$

And finally, it is more than simple exercise to work out the kinetic energy of mass m by using (12) above:

$$K = (\gamma - 1) \ mc^2 = \frac{mc^2}{\sqrt{1 - [\frac{M^2 - m^2}{M^2 + m^2}]^2}} - mc^2 \ . \tag{13}$$

This is the required answer.

Problem 4

Here an incident photon of light is absorbed by an electron. As a result, the electron gains energy. Now to eject the electron, the photon must provide two kinds of energy: (1) An energy necessary for the electron to overcome the forces holding it inside the metal structure (something that is called the work function of the metal, denoted ϕ), and (2) An excess energy which manifests itself as the kinetic energy of the electron. Thus energy conservation reads the following:

$$\frac{hc}{\lambda} = \phi + K \ . \tag{14}$$

Now if the requirement is just to barely free the electron, then $K = 0$ above and the required photon energy is a minimum:

$$\frac{hc}{\lambda}|_{min} = \phi \ \Rightarrow \ \frac{hc}{\lambda_{max}} = \phi \ \Rightarrow \ \lambda_{max} = \frac{hc}{\phi} \ . \tag{15}$$

Note that the minimum photon energy corresponds to the maximum wavelength. Finally, let us calculate this wavelength:

$$\lambda_{max} = \frac{hc}{\phi} = \frac{2\pi\hbar c}{\phi} \approx \frac{(2\pi)(200 \text{ Mev.fm})}{(2 \times 10^{-6} \text{ Mev})} \approx (\pi)(2 \times 10^8 \text{ fm}) \approx 6 \times 10^8 \text{ fm}$$

$$= 6 \times 10^{-5} \text{ cm} = 6000 \text{ A} \ . \tag{16}$$

This is the required answer.