YOUR NAME:__________ ROLL NUMBER:__________

Quiz # 2 Saturday, November 17, 2012

Answers must be given on this sheet of paper only. Use the backsides for rough calculations. No calculators, mobile phones or crib sheets are permitted. Time=90 minutes.

Q.1 Let $|\psi(t)\rangle$ be the state of a particle of mass m that moves in one dimension under the influence of some potential $V(x)$. The Hamiltonian is $\hat{H}=\dfrac{\hat{p}^2}{2m}+V(x)$.

a) How is $|\psi(t)\rangle$ related to $|\psi(0)\rangle$ and $\langle\psi(t)|$ related to $\langle\psi(0)|$? [1+1]

b) The average momentum at time t is $\langle p(t)\rangle=\langle\psi(t)|\,\hat{p}\,|\psi(t)\rangle$. Calculate $\dfrac{d}{dt}\langle p(t)\rangle$. [6]

c) Show that the answer you calculated above is actually Newton's Second Law. [2]

Q.2 In class we dealt with the SHO with $\hat{H}=\frac{p^2}{2m}+\frac{1}{2}kx^2$. In this problem put $m=k=\hbar=1$ (this is not as crazy as you might think!). Define two new operators a and $a^\dagger$ as follows:

$a=\frac{x+ip}{\sqrt{2}}$ and $a^\dagger=\frac{x-ip}{\sqrt{2}}$. (Note that $[x,p]=i$)

a) Show that $[a,a^\dagger]=1$ [2]

b) Show that $\hat{H}$ can be rewritten in terms of a and $a^\dagger$ as $\hat{H}=\frac{1}{\cancel{2}}(a^\dagger a+\cancel{1}\tfrac{1}{2})$. [4]

$\hat{H}=\left(a^\dagger a+\frac{1}{2}\right)$

c) Let $|n\rangle$ be the n'th eigenstate of the operator $\hat{N}=a^\dagger a$, i.e. $\hat{N}|n\rangle=n|n\rangle$. where $n=0,1,2,\ldots$ What are the possible energies of this SHO? [2]

d) If ~~$|\psi\rangle=a^\dagger|n\rangle$ is an eigenstate of $\hat{N}$, show that $|\psi\rangle$ is also an eigenstate of $a^\dagger$ and find the eigenvalue. [2]~~

It is given that $|\psi\rangle=a^\dagger|n\rangle$ is an eigenstate of $\hat{N}$. Find the corresponding eigenvalue.

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial\varphi^2}\right)$$

Q.3 The normalized wavefunction of a hydrogen atom in the ground state is $\frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$.

a) Operate directly on this with $\hat{H}$ and find the energy eigenvalue [5].

b) What is the probability that the electron will be found at a distance greater than 2.5 a_0. [5]

Q.4 A two - level system has a Hamiltonian $H = H_0 + H'(t)$ where,
$H_0 = \varepsilon_1 |1\rangle\langle 1| + \varepsilon_2 |2\rangle\langle 2|$ and $H'(t) = Ve^{i\omega t}|1\rangle\langle 2| + Ve^{-i\omega t}|2\rangle\langle 1|$. where V is a real number. At $t = 0$ the system is in state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$, and at a later time t, $|\psi(t)\rangle = c(t)|1\rangle + d(t)|2\rangle$.

a) Find the equations obeyed by $c(t)$ and $d(t)$. [5 points]

b) Assuming V to be small, find $c(t)$ and $d(t)$. [5 points]