PHYSICS OF THE SUN

1. In this lecture I shall pull together the different parts of physics that you have learned in this course and apply them to understanding the source of all life on earth - our sun. We will learn that the sun operates according to principles that we can understand, and on the basis of this we can even predict the manner in which it will eventually die.

2. Basic solar facts:
   a) Mass of sun $= 2\times10^{30}$ kg = 333,000 Earth's
   b) Diameter of sun $=$ 1,392,000 km = 10^9 Earth's
   c) Age of sun = 4.6 billion years
   d) Rotation Period = 25 days at equator, 36 at poles (surface)
   e) Temperature = 15 million ^{0}K at core, 5770 ^{0}K at surface
   f) Density = $8\times$gold at the core, average is ~ 1.5 water
   g) Composition: 72% H, 25% He, rest is metals


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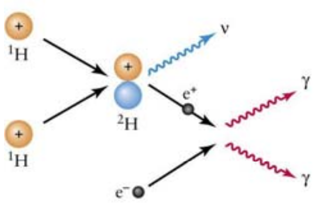
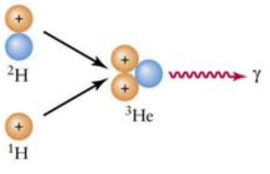
3. The sun puts out a huge amount of energy. In quantitative terms we measure this in by its luminosity, 3.83×10^{27} joules per second. The power output is 3.83×10^{24} kilowatts. This is equal to 8×10^{16} of the largest power plants on Earth, meaning those which produce ~5000 MW of power. Another way of expressing this: every second the sun puts out as much energy as 2.5×10^9 (2 billion) such power plants would put out every year.

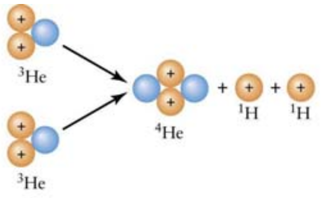
4. What powers the sun? The earth is very old (billions of years). If there was a chemical fuel (say, coal or oil) at most that would last a few million years. But the sun is many thousands of tons older than that. Only after the discovery of $E = mc^2$ did we know the real secret. The sun gets its energy from the fusion (the coming together and combining of atomic nuclei. For this extremely high temperatures, density, and pressure is needed.



a Step 1:
- Two protons (hydrogen nuclei, ^{1}H) collide.
- One of the protons changes into a neutron (shown in blue), a neutral, nearly massless neutrino (ν), and a positron (e^+), a positively charged electron.
- The proton and neutron form a hydrogen isotope (^{2}H).
- The positron encounters an ordinary electron (e^-), annihilating both particles and converting them into gamma-ray photons (γ).

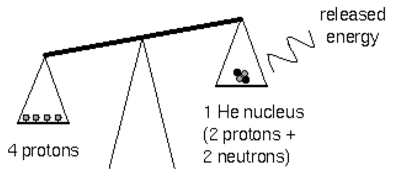


b Step 2:
- The ^{2}H nucleus from the first step collides with a third proton.
- A helium isotope (^{3}He) is formed and another gamma-ray photon is released.



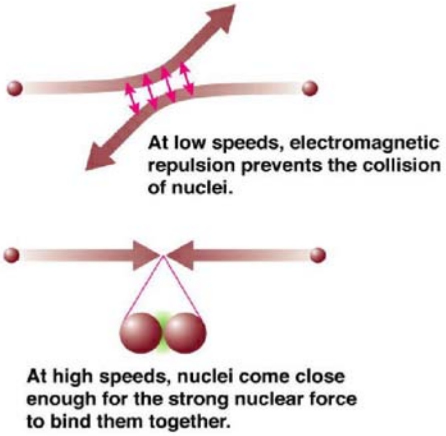
c Step 3:
- Two ^{3}He nuclei collide.
- A different helium isotope with two protons and two neutrons (^{4}He) is formed and two protons are released.

4. The basic point is that, as you can see in the picture, the combined mass of 4 protons is higher than than that of the helium nuclei into which they convert through the process shown earlier. The difference then appears in the form of kinetic energy of the released particles, which in random form is heat.



Some mass is converted into energy (E=mc^2)

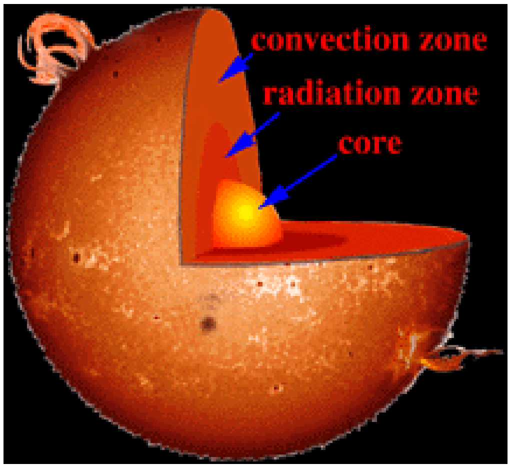
5. Protons repel protons, but the only way in which they will participate in a fusion reaction is when they can come sufficiently close. This requires that they smash into each other at sufficiently high speeds, and hence nuclear fusion in the sun requires core temperatures greater than about 8 million ^{0}K. He nuclei can also fuse with each other to release energy, but because they are heavier much higher temperatures & densities (about 100 million ^{0}K for helium fusion) are required. Thus, stars fuse hydrogen first. Each second, the sun turns 4 million tons of hydrogen into energy.

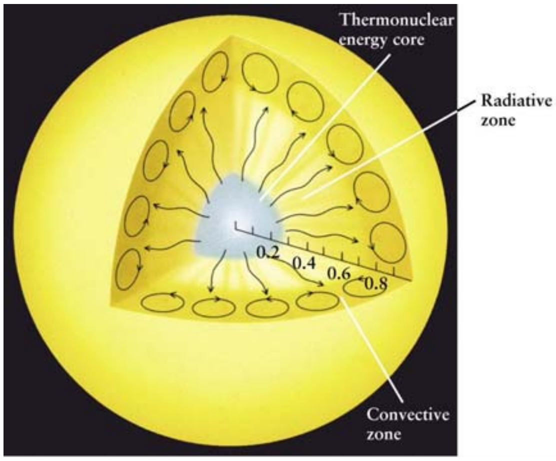


At low speeds, electromagnetic repulsion prevents the collision of nuclei.

At high speeds, nuclei come close enough for the strong nuclear force to bind them together.

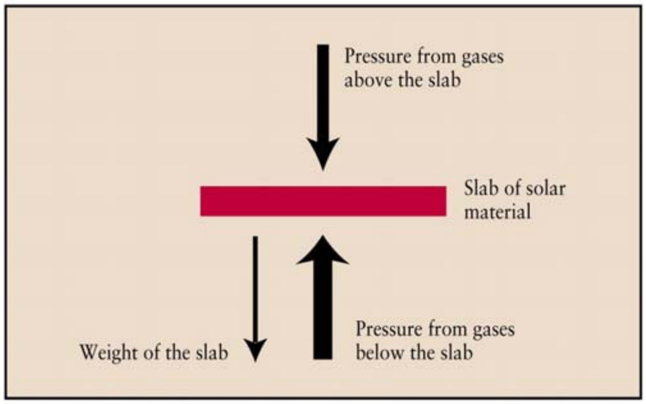
6. Fusion takes place only in the core of the sun (see diagrams below) because it is only there that the hydrogen gas is hot enough. From the core, the heat gets out by the emission of photons (radiation zone). The hot gas then exchanges heat with the sun's relatively cold exterior through convective currents. Huge columns of hot gas move from the inside towards the surface. After giving up most of the heat, the "cold" gas sinks towards the centre and the cycle goes on.

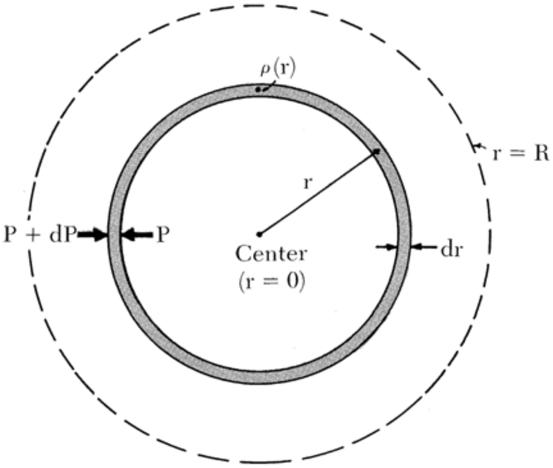




7. For over 4 billion years the sun has been nearly steady in maintaining its size. It is in a state of equilibrium between two big forces acting oppositely to each other:
   a) The hot hydrogen gas seeks to expand outwards because the thermal velocity of H atoms leads to a pressure directed outwards.
   b) Gravity tries to squeeze the star inwards because every piece of matter attracts every other piece. So the gravitational pressure is inward, increasing toward the core.

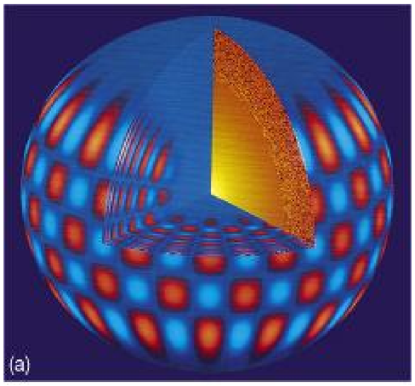
   As in the diagrams below, imagine a piece of solar material at some distance from the centre of the sun. The two forces acting upon it must exactly balance in equilibrium. If for some reason the sun cools down, the pressure of gas will decrease and the sun will contract to a new equilibrium point. Conversely, the sun will expand if the rate of fusion were to increase.

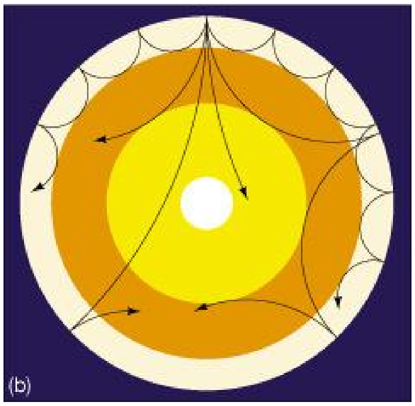




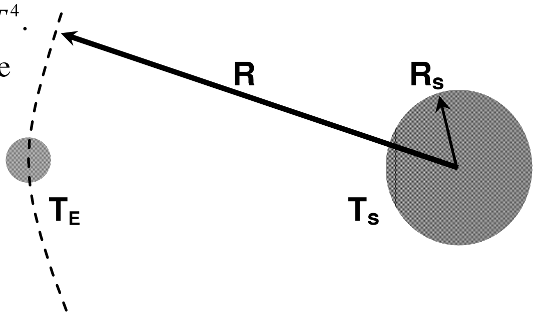
Look at the second diagram. Let $M(r)$ be the mass contained within radius r. We will not assume that the density is constant in r. First find the inward directed gravitational force on a shell of matter at radius r and thickness dr, $dF = -\frac{GM(r)\times \rho(r) 4\pi r^2 dr}{r^2}$. There is a net pressure as shown, and we will call dP the difference of pressures. Then obviously $dF = dP \times 4\pi r^2$. Hence, $dP = -\frac{GM(r)\rho(r)dr}{r^2}$ or, $\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$. The total mass upto radius r is, $M(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$. If we knew $\rho(r)$ then $M(r)$ would also be known. By solving the differential equation, we would also then know $P(r)$. To give us a better understanding, suppose for simplicity that the sun is approximately uniform. Then the density is, $\rho \approx \frac{M}{(4\pi/3)R^3} \approx 1.4 \text{gm cm}^{-3}$. Hence the pressure at the centre of the sun can be computed, $P_{centre} \approx \frac{GM\rho}{R} \approx 3\times 10^9$ atmospheres.

8. The equilibrium of forces is a nearly perfect one, but there are small disturbances and these cause "earthquakes" (actually sun-quakes) to occur on the surface. The study of the surface of the sun and sun-quakes is an area known as "helioseismology" (helio=sun, seismology is the study of vibrations and earthquakes). The pulsating motions of the sun can be seen by measuring the Doppler shifts of hydrogen lines across the face of the sun. Some parts are expanding towards the earth while adjacent regions contract away. This is like the modes of a ringing bell. Vibrations propagate inside the sun, and the waves travel through a hot dense gas. They experience reflection and refraction since the speed at which a wave travels changes when it goes from a region of high to low density or vice-versa. All this is being studied by astrophysicists today as they map out the sun's interior.

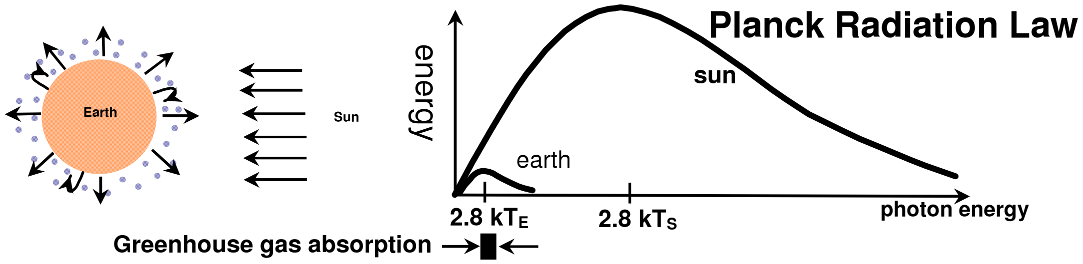




9. Let us calculate the surface temperature of a planet circulating the sun. We shall use the Stefan Boltzman law that you studied in the lecture of heat: the power radiated by a black body per unit surface area at temperature T is σT^4. For thermal equilibrium, we must have that all the power absorbed from the sun is re-radiated by the planet. Let P_S be the sun's flux at its surface, Then, $P_S = \sigma T_S^4$. Since radiation decreases by the distance squared, $P_R = \text{flux at earth} = \sigma T_S^4 \times \left(\frac{R_s}{R}\right)^2$ This must be multiplied by πR_E^2, which is the crossectional area of the planet. On the other hand, for emission from the planet, $P_E = \text{flux at its surface} = \sigma T_E^4$. We must multiply this by $4\pi R_E^2$, the area of planet, to get the total radiated power. Now impose the equilibrium condition: $P_R \times \pi R_E^2 = P_E \times 4\pi R_E^2$. This gives $\sigma T_S^4 \times \left(\frac{R_s}{R}\right)^2 = 4\sigma T_E^4$, and hence $T_E = \left(\frac{R_s}{2R}\right)^{1/2} T_S$. This is true for any planet, so let's see what this gives for the earth using $R_S = 7\times10^8$ metres, $R = 1.5\times10^{11}$ metres, and $T_S = 5800^o K$. This gives $T_E = 280K$, which is very sensible! Of course, we have assumed that the earth absorbs and emits as a black body. True?



10. Actually, the black body assumption is only approximately true. About 30% of the sun's radiation reflects off our atmosphere! In fact the average surface temperature of the earth is about 290K, or about 13 ^{0}C. The extra warming is mainly due to the "greenhouse effect". Certain gases trap the sun's radiation and it is not able to get out, thus leading to greater absorption. The main greenhouse gases are CO_2, CH_4, H_2O. (N_2, O_2 and Ar are transparent to solar and earth radiation.) CO_2 is now 360 ppm (parts per million) of the atmosphere, up from 227 ppm in 1750, before the industrial revolution. Plants turn H_2O + CO_2 into O_2 plus organics. Some estimates predict a 3 to 10 ^{0}C rise in the earth's surface temperature over the next 100 years due to the increased CO_2 greenhouse effect.

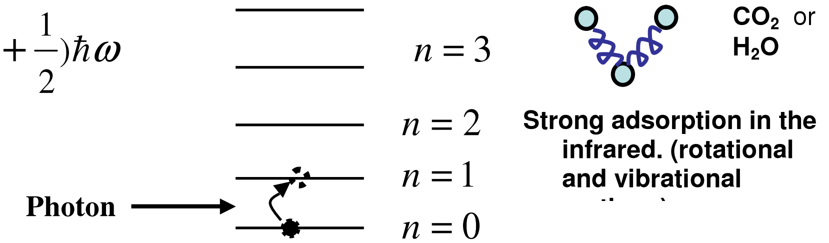


In the above, you can see the narrow window in which the greenhouse gases absorb the sun's radiation. But why are only some gases responsible, and not others?

11. The answer lies in quantum mechanics. In a previous lecture you learned that molecules can exist only in certain states that have very specific values of energy. For example, a molecule of CO_2 can oscillate and have equally spaced energy levels as shown below:

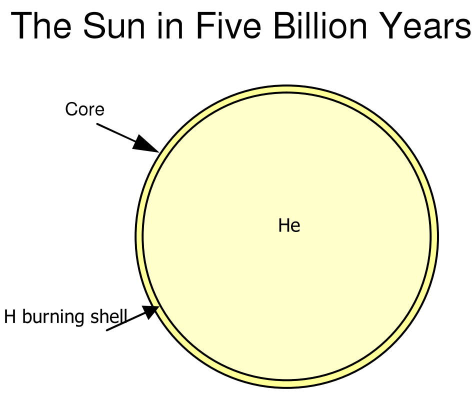
Energy levels of an oscillator:

$$\varepsilon_n = (n + \frac{1}{2})\hbar\omega$$

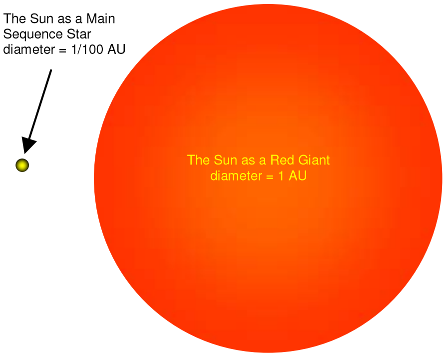


The energy at which these molecules can absorb radiation happen to lie in the spectrum of the re-radiated energy from the earth's surface. Thus, they effectively trap the outgoing radiation, leading to enhanced earth temperatures. For any molecule, we can both calculate (using quantum mechanics) the frequencies at which radiation is absorbed or emitted. We can also experimentally measure the frequencies. Both of these are in very good agreement, and this is one of the reasons why we have such confidence in the correctness of quantum mechanics.

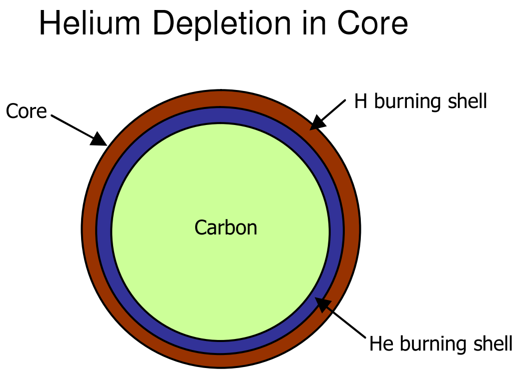
12. Finally, I have some bad news: the sun is going to die because the hydrogen supply is eventually going to run out. There is, of course, no immediate danger - the Sun can last another 5 billion years on core hydrogen fusion. During this time it will be consuming hydrogen and producing helium. At the end of this phase, the core of the sun will have mostly helium with the little bit of hydrogen left almost entirely outside in a thin layer. The reaction rate will fall, and the core will no longer be able to balance the pull due to gravity. This will cause a shrinkage of the core. As a consequence the temperature will increase. A new phase of the sun is about to start.



13. At this point a fusion reaction in next core zone begins. This lifts the envelopes and the sun will brighten up significantly. It is now a Red Giant star, and as you can see below, it is huge! The earth will be swallowed up by the expanded sun.



14. As remarked earlier, helium can also "burn" to produce carbon, but it needs a much higher temperature. Eventually even the He will be depleted, and the reaction rate will fall. Again the small reaction rate means that the core will shrink, and heat up to the point that it crosses the carbon fusion threshold of 600 million K. Low mass stars cannot reach this temperature. Envelope expands to a supergiant, many times larger than even a Red Giant.



QUESTIONS AND EXERCISES

Q.1 Calculate the power radiated by a 10-cm-diameter sphere of metal at room temperature (20 ^{0}C). Give your answer in watts.

Q.2 a) Why is the rotation period of the sun different at the equator as compared to that at the poles?

b) How do we know for sure that the centre of the sun is much hotter than the surface?

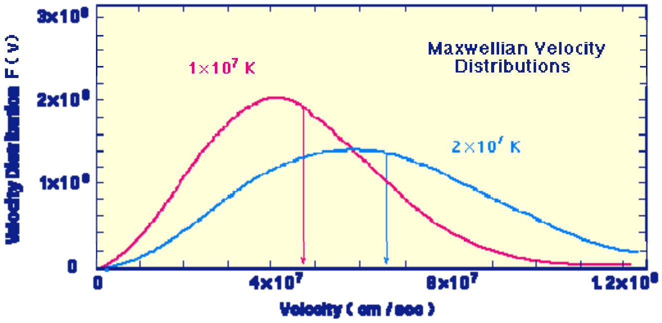
c) Since it is impossible to go near the sun, how do we know the chemical composition of the sun? Its surface temperature?

Q.3 In a previous lecture you learned how to calculate the average speed of atoms in a hot, ideal gas.

a) Calculate the average speeds at $T = 10^7\ {}^0K$ and $T = 2\times 10^7\ {}^0K$. Compare with the average estimated from the graph below for hydrogen nuclei (protons) in a star.

b) Protons repel each other. Suppose two protons collide head-on with each other at some speed v. What will be the closest they will come to each other.

c) For typical speeds in the graph below, estimate this distance. Will this be sufficient for a nuclear reaction to take place?



4. Refer to the derivation of the solar pressure-distance relation: $\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$. This was solved assuming a constant density. In this problem, suppose we assume a linearly decreasing density, $\rho(r) = \rho_0(1 - r/R)$, where R is the solar radius.

a) Find $M(r)$ and make a plot.

b) Solve $\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$. This is a first-order differential equation and so will have one arbitrary constant. How should it be fixed? Is it sensible to have $P(R) = 0$?