LUMS School of Science and Engineering

Modern Physics Midterm Examination

Instructor:Dr. Pervez Hoodbhoy 20 October 2012 Time =2 hours 30 minutes

No papers/books/cell-phones/calculators are allowed

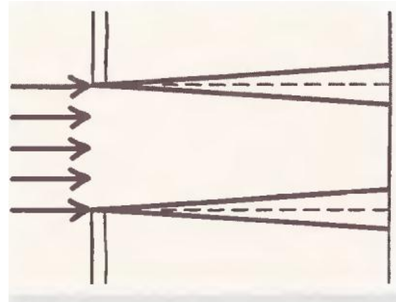
Answer all questions on this sheet.

Q.1 a) A neutron moves freely inside a large nucleus of size 10fm. Taking the mass of the neutron to be approximately 1000 MeV/c^2 (it is actually 939.35 MeV/c^2), find its kinetic energy. [5]

b) An electron is trapped in a potential well of size 10A. What is the smallest amount of energy it can have? [5]

Q.2 A photon with frequency ν moves in the $\hat{x}$ direction and hits a stationary atom of mass M. It moves off in the $\hat{y}$ direction with frequency ν'. By writing down the conservation of momentum and energy, find ν'. [10]

Q.3 A wide beam of electrons of momentum p impinges upon a slit of width a. If these were classical bullets instead of electrons, the width of the screen image, which is at distance D would be exactly a.



a) Using the uncertainty principle, find the width of the screen image for electrons. [7]

b) For what value of a does the screen image width have a minimum for electrons? Discuss. [3]

Q.4 Suppose that $|\uparrow\rangle$ and $|\downarrow\rangle$ are the two orthonormal states representing a spin-up and spin-down electron respectively. Each of 10^6 electrons are somehow prepared in the state $|\psi\rangle$,

$$|\psi\rangle = \sqrt{\frac{4}{5}}|\uparrow\rangle + i\sqrt{\frac{1}{5}}|\downarrow\rangle.$$

They are then passed through a Stern-Gerlach apparatus.

a) What do you expect will be the net number of spins up minus the number down? [2]

b) After an electron has passed through the apparatus, will it still be in the state $|\psi\rangle$ above? If not, then what could be its state after the measurement? [2]

c) The operator $\hat{R}$ is such that, $\hat{R}|\uparrow\rangle = \cos\theta|\uparrow\rangle + \sin\theta|\downarrow\rangle$, $\hat{R}|\downarrow\rangle = -\sin\theta|\uparrow\rangle + \cos\theta|\downarrow\rangle$.

Find $|\varphi\rangle = \hat{R}|\psi\rangle$. [3]

d) Find $\langle\psi|\varphi\rangle$. Interpret your result. [3]

Q.5 An electron is constrained to move along the x axis. Its wavefunction is $\psi(x) = Ne^{-\frac{1}{2a}|x|}e^{-i\omega t}$.

a) Find the normalization constant N. [5]

b) What is the chance of finding the electron between $x = 1$ and $x = \infty$? [5]

Q.6 The fraction P_n of particles of mass m and energy E that tunnel through a barrier of (small) thickness Δx_n and potential energy $U(x_n)$ is,

$$P_n = e^{-\frac{2}{\hbar}\Delta x_n \sqrt{2m(U(x_n)-E)}}.$$

Suppose that a sequence of barriers are adjacent to each other at positions $x_1, x_2, \cdots$ have potential energies $U(x_1),\ U(x_2), \cdots$ and the respective fractions are $P_1,\ P_2, \cdots$. Since the particle penetrates the barriers individually, the total penetration probability for N barriers is $P = P_1 P_2 \cdots P_N$. Show that in the limit where each barrier becomes very thin, the total probability for an arbitrary shaped potential barrier between $x = a$ and $x = b$ is,

$$P = e^{-\frac{2}{\hbar}\int_a^b dx \sqrt{2m(U(x_n)-E)}}. \quad [10]$$