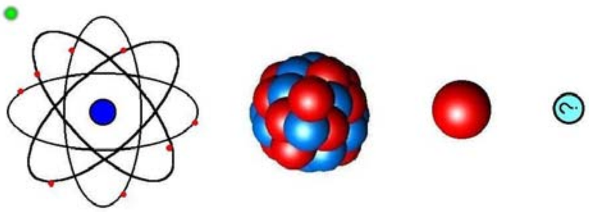
INTRODUCTION TO NUCLEAR PHYSICS

Q.1 In the previous lecture you learned how it was discovered that the atom is mostly empty space with a cloud of electrons. At the centre is a small but very heavy nucleus that has protons and neutrons. The word "nucleon" refers to both of these. So you can think of the neutron or proton as being two different varieties of the nucleon. The masses of the two are very similar, and they are roughly 2000 times heavier than the electron.



$$\text{proton mass} = M_p = 1.672\times10^{-27}\,kg$$

$$\text{neutron mass} = M_n = 1.675\times10^{-27}\,kg$$

$$\text{electron mass} = M_e = 9.109\times10^{-31}\,kg$$

The neutron is neutral, of course, but the charge on the proton is $1.6\times10^{-19}C$ while the charge on the electron is the negative of this, $-1.6\times10^{-19}C$.

2. Using kilograms is very awkward if you are dealing with such small particles. Instead we we use $E = mc^2$ to write the mass of a particle in terms of its rest energy, $m = E/c^2$. So mass is measured in units of MeV/c^2.

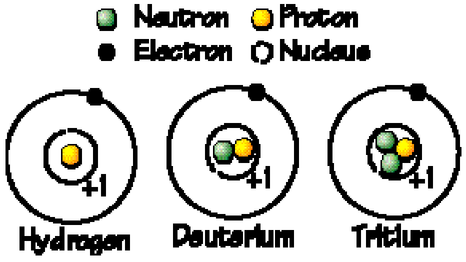
$$\text{proton mass} = M_p = 938\,MeV/c^2$$

$$\text{neutron mass} = M_n = 940\,MeV/c^2$$

$$\text{electron mass} = M_e = 0.5\,MeV/c^2$$

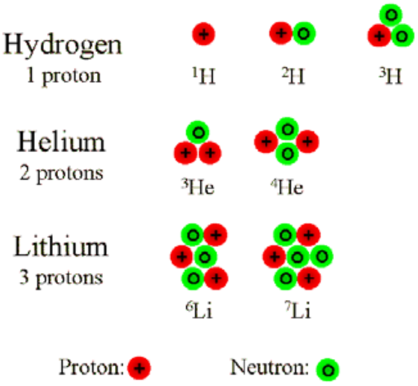
3. a) A hydrogen nucleus is just one proton.
   b) A deuteron has a proton plus one neutron.
   c) A triton (or tritium nucleus) has a proton plus two neutrons.

   All three atoms have one electron only, and thus completely identical chemical properties.

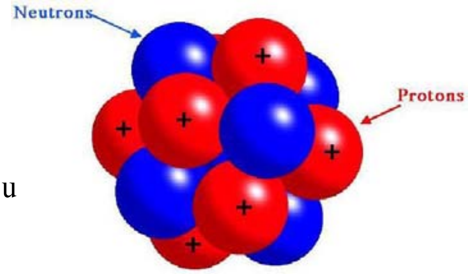


4. Hydrogen, deuterium, and tritium are called isotopes. If a nucleus with Z protons has N neutrons then its isotopes will have fewer, or more, neutrons. Since the number of electrons is also Z, the chemical properties of all isotopes are exactly the same. But for any given element, at most there is only one stable isotope.

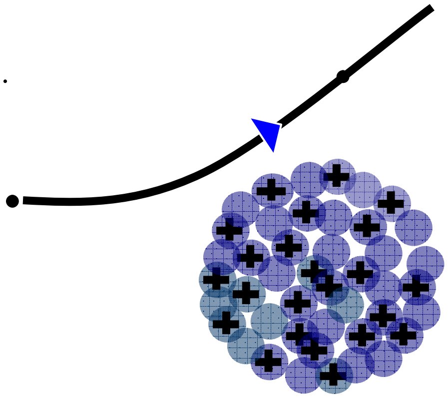
4. A commonly used notation is ${}^A_Z X$ where X is the element, and $A = Z + N$. Of the elements that you see on the right, the most stable ones are ^{1}H, ^{3}He, and ^{6}Li. Now consider oxygen. The most stable isotope is ^{16}O. When you breathe in oxygen from the atmosphere, 99.8% is ^{16}O, 0.037% is ^{17}O, and 0.163% is ^{18}O. We shall see later what unstable means and how nuclei decay.



5. The diameter of the nucleus is about 10 million times smaller that the overall diameter of the atom. Nuclei follow an approximate rule for the radius, $r \approx r_0 A^{1/3}$ where $r_0 = 1.2\ fm$ (remember, 1 fermi $= 10^{-13}$ cm) and $A = Z + N$. Now, $A^{1/3}$ increases very slowly with A. You can check that $16^{1/3} = 2.52$ while $208^{1/3} = 5.93$. This means that a very heavy lead nucleus ^{208}Pb is only about 2.4 times the size of the much lighter ^{16}O nucleus.

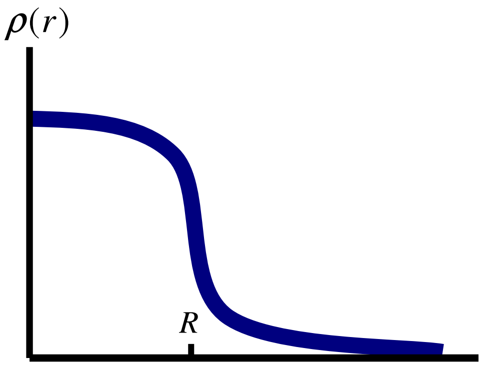


6. To learn about how protons are distributed inside a nucleus, we send a beam of electrons at a nucleus and observe how they scatter in different directions. The negatively charged electrons interact with the positively charged protons, but they obviously will not see the neutrons. The scattered electrons are captured in a detector which can be moved around to different angles. In this way one can reconstruct the charge distribution which caused the electrons to be scattered in that particular way.



7. What energy should electrons have in order to see a nucleus? We know that electrons are waves with $\lambda = h/p$ (the De Broglie relation). To see something as small as $1 fm$ requires a wave with wavelength at least $\lambda \approx 1 fm$. A wave with longer wavelength would simply pass over the nucleus without being disturbed. So the minimum electron energy is, $E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$. Evaluation gives this to be a few MeV, requiring an electron accelerator of more than this minimum energy.

8. From electron scattering we see that the proton density is almost constant throughout the nucleus and falls sharply at the surface. So nuclei should be thought of as rather fuzzy balls. A typical plot of density versus distance from the centre looks like this. Here the distance $R \approx r_0 A^{1/3}$ can be called the nuclear radius. The distribution of neutrons is very similar to this plot, but requires other techniques.

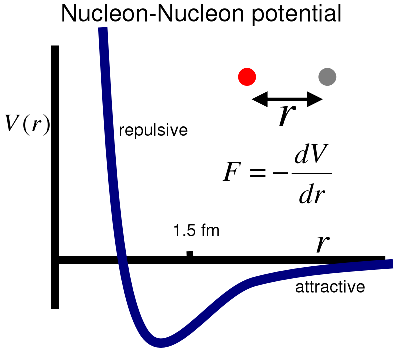


9. Let us consider the implication of the approximate formula for the nuclear radius, $r \approx r_0 A^{1/3}$ where $r_0 = 1.2\ fm.$ The volume of the nucleus is: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_0{}^3 A$. From this $\frac{A}{V} = \frac{3}{4\pi r_0^3} \approx 0.14$ nucleons/fm^3. This is the number of nucleons per cubic fermi, and is independent of the nucleus considered. This is the density you would find at the centre of any nucleus. Of course, this is approximately true only but it is quite remarkable.

10. Protons repel protons through the electrostatic force. So why does the nucleus not blow apart. Obviously there must be some attractive force that is stronger than this repulsion. It is, in fact, called the strong force. From what we have learned so far, we can guess some of its important features:
    a) Since neutron and proton distributions are almost the same, the N-P force cannot be very different from the N-N or P-P force.
    b) Since the density of nucleons in large nuclei is the same as in lighter nuclei, this means that a given nucleon feels only the force due to its immediate neighbours, and does not interact much with nucleons on the other side of the nucleus. In other words, the range of the nucleon-nucleon force is very short and of the order of 1-2 fm only.

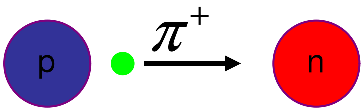
11. The force between two charges is always of one sign - repulsive if the signs are the same, and attractive if they are opposite. In the early years of quantum theory, people realized that this force comes about because of the exchange of photons between charges. The nucleon-nucleon force is different. It has to be attractive to keep the nucleus together, and has to short range (as discussed above). But, to prevent nucleons from sticking to each other, it must be repulsive at short distances. Now here, "short" and long means distances on the scale of fermis. Typically the distances between nucleons is on this scale as well.

12. Here is roughly what the N-N potential looks like. Since the force is the negative of the slope, you can see that for large value of r, the force is attractive (negative means directed towards smaller values of r). At roughly $1.4\,fm$ the potential reaches its most attractive point. For values of r smaller than this, the force is repulsive. In fact there is a very strong core that almost completely forbids the nucleons from getting closer than about $0.5\,fm$.

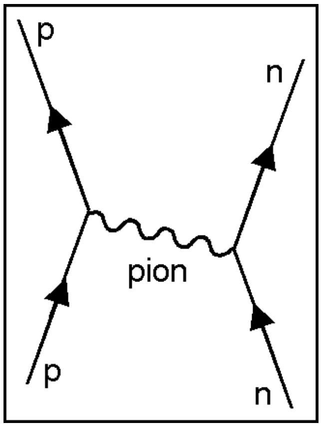


12. In 1935, the Japanese physicist Hideki Yukawa made an astonishing breakthrough in understanding the basis for the attractive N-N force. He assumed that, just as between charges, there must be a particle that is emitted by one nucleon and then captured by the other. He called this a "pi-meson" or "pion", and made a good guess for what its mass should be. His argument uses the time-energy uncertainty principle discussed earlier:

- Let Δt be the time that the pion takes between emission and capture. In this time it will have travelled a distance approximately equal to $c\Delta t$ because light particles can travel no faster than light.
- Creating a pion from "nothing" means that energy conservation has been violated. The amount of violation is $\Delta E =$ minimum energy of pion $\approx mc^2$ and this must obey $\Delta E\,\Delta t \approx \hbar/2$. Hence, $\Delta t \approx \frac{\hbar}{2mc^2}$ and the distance travelled by the pion $\approx$ c $\Delta t \approx \frac{\hbar}{2mc}$.



- Put $\frac{\hbar}{2mc} \approx 1.2\,fm$ (range of nuclear force). This gives the mass of the pion as close to $mc^2 \approx 124\,MeV$. This was an amazing prediction - the first time a particle had been predicted to exist on the basis of a theoretical argument. When experimentalists searched for it in 1947, they indeed found a particle of mass rather close to it, with $mc^2 \approx 138\,MeV$. It was a very dramatic confirmation for which Yukawa got the Nobel prize.



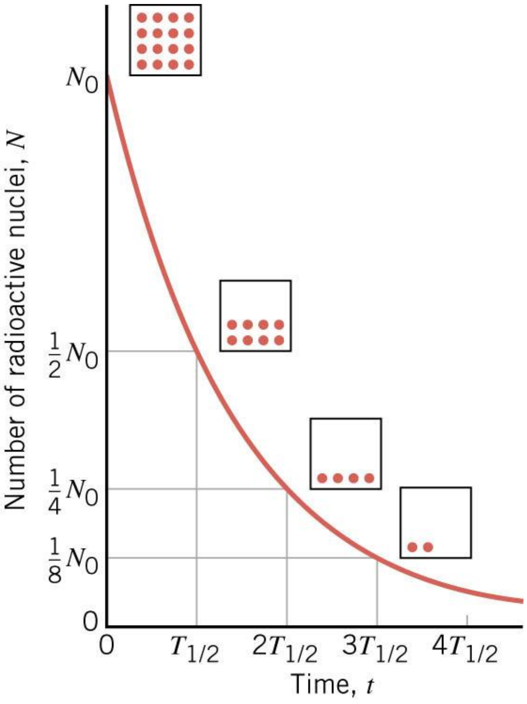
- Pions can rightfully be called the carriers of the strong nuclear force. They have 3 possible charge states: π^+, π^0, π^-. They belong to a larger family of particles called mesons. Other family members are rho-mesons, omega-mesons, K-mesons,... Today we can produce mesons in huge amounts by smashing nucleons against each other in an accelerator.

13. A few nuclei are stable, most decay. The decay law is simply derived: if the number of nuclei decreases by dn in time dt, then dn must be proportional to both the the number of nuclei n that are decaying and dt, so $dn \propto -ndt$ (minus sign for decrease). with the proportionality constant λ, we have $dn = -\lambda ndt$, or $\frac{dn}{dt} = -\lambda n$. We have encountered the solution of this type of equation before, $n = n_0 e^{-\lambda t}$. You can see that at $t = 0$, $n = n_0$. Taking the log, we have $\ln \frac{n}{n_0} = -\lambda t$. We define the half-life $T_{\frac{1}{2}}$ as the time it takes for half the original sample to decay. If $n = n_0/2$ then $\log \frac{n_0}{2n_0} = -\lambda t$, from which the half life is related to λ by, $T_{\frac{1}{2}} = \frac{\log 2}{\lambda} = \frac{0.693}{\lambda}$. The larger λ, the more radioactively unstable a nucleus is. Some typical half-lives are:

Polonium	$^{214}_{84}P$	$1.64\times 10^{-4}\,s$
Krypton	$^{89}_{36}$K	3.16 minutes
Strontium	$^{90}_{38}$Sr	28.5 years
Radium	$^{226}_{88}$Ra	1600 years
Carbon	$^{14}_{6}$C	5730 years
Uranium	$^{238}_{92}$U	4.5×109 years

You can see how hugely different the lifetimes of different nuclei are!

14. Here is a plot of the number of unstable nuclei left as a function of time. After each half-life, the number of nuclei decreases in number by half of the previous. Eventually there is only one nucleus left, and that too will eventually decay. So how can the derivation for the decay law be correct? Strictly speaking, we are not allowed to write down, or solve, a differential equation like $\frac{dn}{dt} = -\lambda n$ because this assumes that $n(t)$ is a continuous function. But this is almost true because in real life we deal with very large numbers of nuclei and so it makes a lot of sense to think of $n(t)$ as continuous.



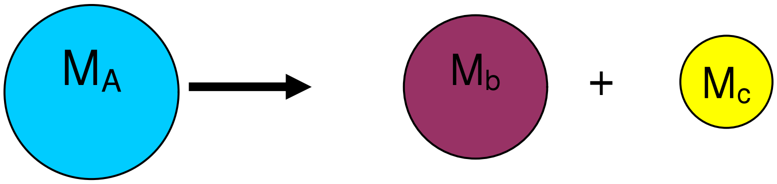
15. Just to get an idea, consider the decay of $^{222}_{86}\mathrm{Rn}$ (Radon, a very dangerous gas that is found underground) into $^{218}_{84}\mathrm{Po}$ (Polonium, another terrible poison) and $^{4}_{2}\mathrm{He}$ (harmless, fortunately!). The half life is 3.8 days. So, if we started with 20,000 atoms of $^{222}_{86}\mathrm{Rn}$, then in 3.8 days we would have 10,000 atoms of $^{222}_{86}\mathrm{Rn}$ and 10,000 atoms of $^{218}_{84}\mathrm{Po}$
In 7.6 days we would have 5000 atoms of $^{222}_{86}\mathrm{Rn}$, in 11.4 days, 2500 , $^{222}_{86}\mathrm{Rn}$ etc.

16. The decay law can be used to see how old things are. This is called radioactive dating. Carbon dating is widely used for living things that died a few hundred or few thousand years ago. How does it work?This uses the decay of the unstable isotope, $^{14}_{6}C$. Of course, the stable isotope of carbon is $^{12}_{6}C$.
    - When a living organism dies, $\mathrm{CO_2}$ is no longer absorbed. Thus the ratio of carbon 14:12 decreases by half every 5730 years. We can measure the rate of decrease through $N = N_o e^{-\lambda t}$ or the "activity" $\mathrm{A} = A_o \; e^{-\lambda t}$ with $A_o \; = \; 0.23 \; Bq/g$. (The becquerel Bq is the unit of radioactivity, defined as the activity of a quantity of radioactive material in which one nucleus decays per second.)
    - The amount of isotopes in the atmosphere is approximately constant, despite a half-life of 5730 y because there is a constant replenishment of $^{14}_{6}C$ through the reaction, $^{14}_{7}N + n \rightarrow {}^{14}_{6}C + p$

17. Let us use the above idea to find the time when this man died. His body was found a few years ago buried under deep snow in a mountain pass, so it it did not decay as usual. By looking at the radioactivity in his body, it was found that that the activity of $^{14}_{6}C$ was $0.121 \; Bq/g$ of body tissue. This is less than the normal activity $0.23 \; Bq/g$. because $^{14}_{6}C$ has been decaying away. First find λ, $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{5730} = 1.21 \times 10^{-4} \quad y^{-1}$
Then use, $0.121 = 0.23 \;\; e^{-1.21 \times 10^{-4} \times t}$ which gives,
$\ln \dfrac{0.121}{0.23} = -1.21 \times 10^{-4} \times t$ and so $t \;=\; 5300$ years is when this poor man was killed (or died somehow)!

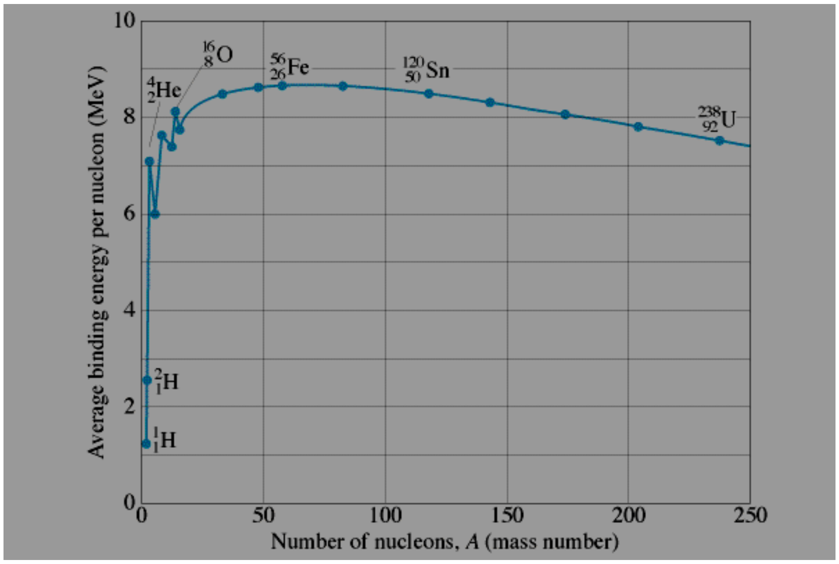
18. The most famous formula of physics, $E = mc^2$, is the basis for nuclear energy. In 1935, it was discovered by two German physicists, Otto Robert Frisch and Lise Meitner, that a heavy nucleus can fission (or break up) into two or more smaller nuclei. The total energy is, of course, conserved but the mass is not. This is completely different from the

usual situation. In the picture below you see an example of fission.



The masses of the two nuclei add up to less than the mass of the parent nucleus, and the energy released is $Q = (M_A - M_b - M_c)c^2$. This goes into kinetic energy and sends the two daughter nuclei flying apart at a large velocity. There happen to be NO completely stable nuclei above $Z = 82$, and no naturally occurring nuclei above $Z = 92$. Above these limits the nuclei decay or fall apart in some fashion to get below these limits.

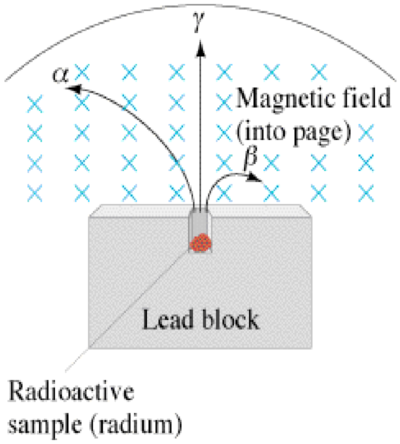
19. A very useful concept is "binding energy". Suppose you want to take a nucleon out of a nucleus. The binding energy is the amount of energy that you would have to provide to pull it on the average. Nuclei with the largest BE per nucleon are the most stable. As you



can see from the graph below, the most stable element is iron, ^{56}Fe with a BE per nucleon of about 8.6 MeV. This is why iron is the heavy element found in the largest quantity on earth and inside stars. The curve is not smmoth and you see that a ^{4}He nucleus (i.e. an α parrticle, has a relatively high binding energy and so is relatively stable. In contrast, Uranium ^{235}U or deuterium ^{2}H are much less bound and they decay.

20. Nuclei can be unstable in different ways. A nucleus can emit $\alpha, \beta,$ and γ radiations. Usually a nucleus will emit one of these three, but it is possible to emit two, or even all three of these. (In addition, as we have discussed above, a nucleus can break up into

two or more smaller fragments through the process of fission.) What is the nature of α, β, γ radiations? In the experiment shown here, you see that a piece of radium has been put in a lead block (as shielding). A magnetic field bends the charged particles emitted during the decay. We find that they are of three types:



a)Heavy positively charged particles are bent to one side by the magnetic field. The amount of bending shows that they are α particles.

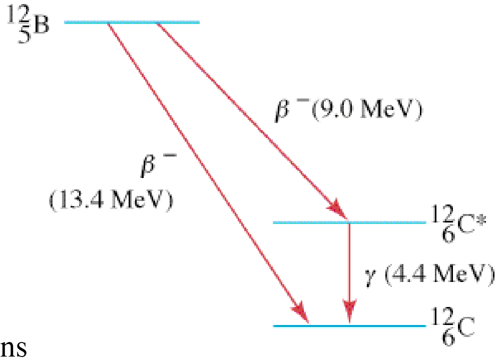
b)Other particles are bent much more, and in the other direction. They are electrons (β particles).

c)Some particles are not bent at all, hence must be neutral. They are γ particles (or rays, or γ photons, same thing!).

21. Let's first consider alpha decay, ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X' + {}^{4}_{2}He$. As you see A changes by 4 and Z by 2. You can think of α particles as a gang of 4 particles that always stays together in a big nucleus. Unstable nuclei simply cannot overcome the proton repulsion and an α particle ultimately succeeds in esaping the nucleus. indefinitely.

22. The simplest beta decay reaction is when a neutron decays, $n \rightarrow p + e^- + \bar{\nu}$. Other than a proton and electron, an anti-neutrino is also emitted. As discussed in the lecture, the (anti) neutrino is a neutral particle with a very tiny mass that interacts very weakly with matter. A nucleus can undergo beta decay with either an electron being produced, i.e ${}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}X' + e^- + \bar{\nu}$ (called β^- decay) or with an anti-electron (positron, or positive electron), ${}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}X' + e^+ + \nu$, (called β^+ decay). Beta decay involves the weak nuclear force. This is one of the four fundamental forces in the world, and its small strength means that the decay happens much more slowly than most other reactions.

23. Just as for an atom, a nucleus can only exist in certain definite energy states. When a nucleus goes from one state to the other, it can emit a photon (γ ray). Because the spacing between nuclear levels is of the order of one MeV (i.e. a million times more than in atoms), the photon is much more energetic than an optical photon. In going between levels, β emission also happens as can be seen in this diagram.

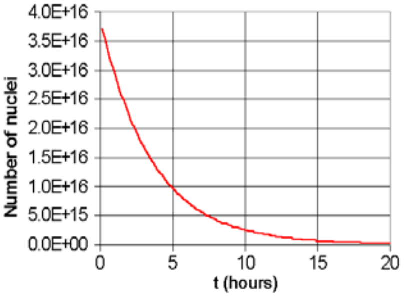


QUESTIONS AND EXERCISES

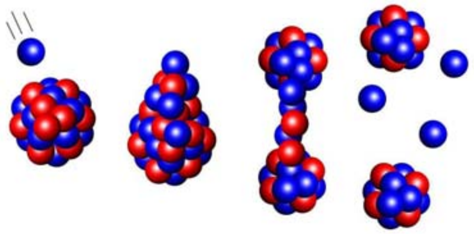
Q.1 In a lab there is 2 micrograms of pure ${}_{14}^{31}\mathrm{Si}$ which has a half life of roughly 2.5 hours. How many nuclei are present initially, and how many are left after 4 hours?

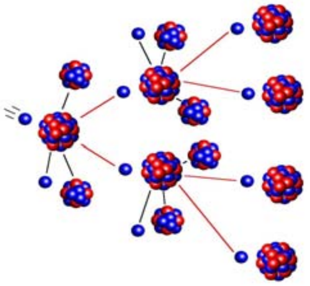
Q.2 The method of carbon dating discussed in the text can only be used for time scales of a few thousand years. For longer time scales we must use the decay of ${}_{92}^{238}U$, which has a half-life of 4.5×10^9 years. Discuss how this could be possible, and how we could use this to find the age of rocks that formed when the earth was formed. You should consult a book or use Google.

Q.3 From the graph shown here, estimate the lifetime of the element. After how much time will the number of nuclei decrease to 4×10^{10}? If you want to use this element for estimating the age of something, in what range of values would it be useful?



Q.4 In the early days of nuclear physics it was discovered that slow neutrons can be very effective in producing the fission of uranium. Typically a reaction could be like this one: ${}_0^1n + {}_{92}^{235}U \to \left({}_{92}^{236}U\right) \to {}_{56}^{141}Ba + {}_{36}^{92}Kr + 3{}_0^1n$. The 3 neutrons produced can then produced more fissions, and this leads to a chain reaction of the kind shown in second diagram:





a)Suppose that a slow neutron enters a block of ${}_{92}^{235}U$. Assuming that all 3 neutrons go on to fission another nucleus. This is called a chain reaction. After 2,3,4,5..fissions, what will be the number of neutrons? Write down a general formula for N fissions.

b)It is possible that some neutrons are too fast and can escape the uranium block without causing fission. Suppose the probability that a neutron does cause fission is p. How is your answer above modified. Evaluate for $p = 0.7$

5. In a collapsed star called a white dwarf, nuclei are almost touching each other. Calculate the density of this matter in kg/m^3 and compare with that of water.