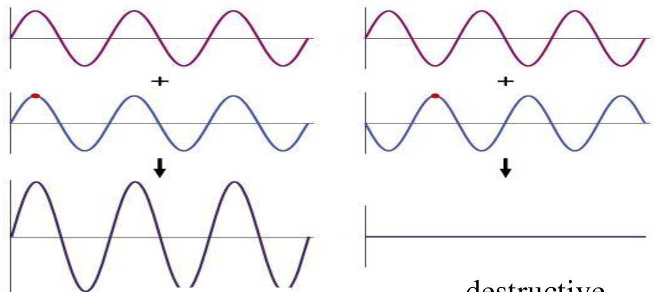
INTERFERENCE AND DIFFRACTION

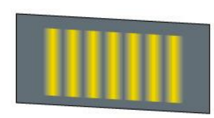
1. Two waves (of any kind) add up together, with the net result being the simple sum of the two waves. Consider two waves, both of the same frequency, shown below. If they start together (i.e. are *in phase* with each other) then the net amplitude is increased. This is called *constructive interference*. But if they start at different times (i.e. are *out of phase* with each other) then the net amplitude is decreased. This is called *destructive interference*.

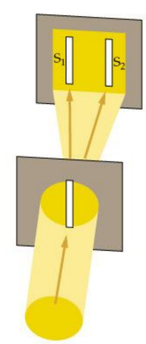


constructive destructive

In the example above, both waves have the same frequency and amplitude, and so the resulting amplitude is doubled (constructive) or zero (destructive). But interference occurs for any two waves even when their amplitudes and frequencies are different.

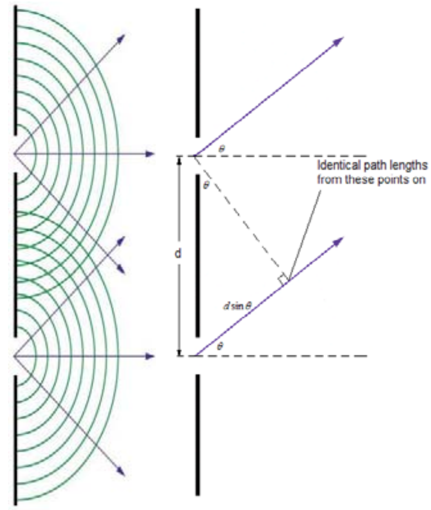
2. Although any waves from different sources interfere, if one wants to observe the interference of light then it is necessary to have a *coherent* source of light. Coherent means that both waves should have a fixed phase relative to each other. Even with lasers, it is very difficult to produce coherent light from two separate sources. Observing interference usually requires taking two waves from a single source, with each going along a different path. In the figure, an incoherent light source illuminates the first slit. This creates a uniform and coherent illumination of the second screen. Then waves from the slits S_1 and S_2 meet on the third screen and create a pattern of alternating light and dark fringes.



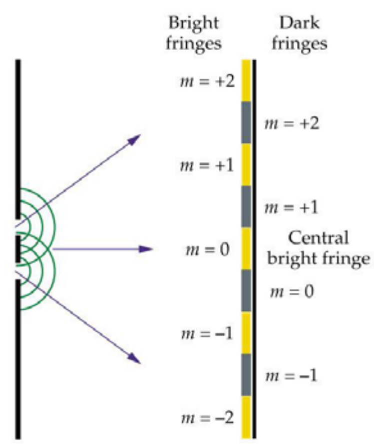


3. Wherever there is a bright fringe, constructive interference has occured, and wherever there is a dark fringe, destructive interference has occured. We shall now calculate where on the third screen the interference is constructive. Take any point on the third screen. Light reaches this point from both S_1 and S_2, but it will take different amounts of time to get there. Hence there will be a phase difference that we can calculate. Look at the diagram below. You can see that light from one of the slits has to travel an extra

distance equal to $d\sin\theta$, and so the extra amount of time it takes is $(d\sin\theta)/c$. There will be constructive interference if this is equal to $T, 2T$, $3T,\cdots$ (remember that the time period is inversely related to the frequency, $T = 1/\nu$, and that $c = \lambda\nu$). We find that $\frac{d\sin\theta}{c} = nT = \frac{n}{\nu} \quad \Rightarrow \quad d\sin\theta = n\lambda$, where $n = 1, 2, 3, \cdots$ What about for destructive interference? Here the waves will cancel each other if the extra amount of time is $\frac{T}{2}, \frac{3T}{2}, \frac{5T}{2}, \cdots$ The condition then becomes $d\sin\theta = (n+\frac{1}{2})\lambda$.



4. Example: two slits with a separation of 8.5×10^{-5}m create an interference pattern on a screen 2.3m away. If the $n = 10$ bright fringe above the central is a linear distance of 12 cm from it, what is the wavelength of light used in the experiment?

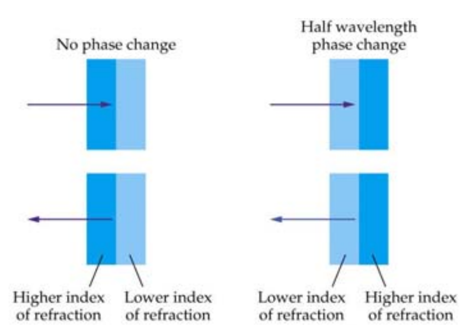


Answer: First calculate the angle to the tenth bright fringe using $y = L\tan\theta$. Solving for θ gives,

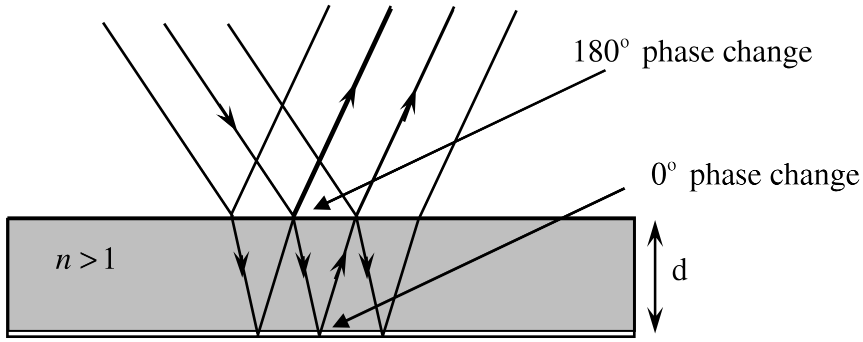
$\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{0.12m}{2.3m}\right) = 3.0^\circ$. From this,

$$\lambda = \frac{d}{n}\sin\theta = \left(\frac{8.5\times 10^{-5}}{10}\right)\sin(3.0^\circ) = 4.4\times 10^{-7}m = 440nm \text{ (nanometres).}$$

5. When a wave is reflected at the interface of two media, the phase will not change if it goes from larger refractive index to a smaller one. But for smaller to larger, there will be phase change of a half-wavelength. One can show this using Maxwell's equations and applying the boundary conditions, but this will require some more advanced studies. Instead let's just use this fact below.



5. When light falls upon a thin film of soapy water, oil, etc. it is reflected from two surfaces. On the top surface, the reflection is with change of phase by π whereas at the lower surface there is no change of phase. This means that when waves from the two surfaces combine at the detector (your eyes), they will interfere.



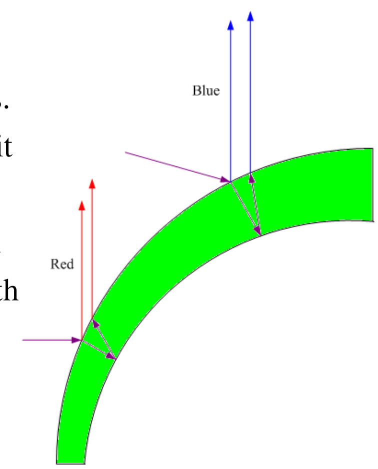
To simplify matters, suppose that you are looking at the thin film almost directly from above. Here n is the index of refraction for the medium. Then,

The condition for destructive interference is: $2nd = m\lambda \quad (m = 0,1,2,...)$

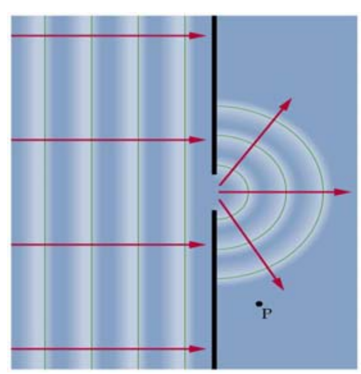
The condition for constructive interference is: $2nd = (m + \frac{1}{2})\lambda \quad (m = 0,1,2,...)$.

Prove it!

6. Interference is why thin films give rise to colours. A drop of oil floating on water spreads out until it is just a few microns thick. It will have thick and thin portions. Thick portions of any non-uniform thin film appear blue because the long-wavelength red light experiences destructive interference. Thinner portions appear red because the short-wavelength blue light interferes destructively.

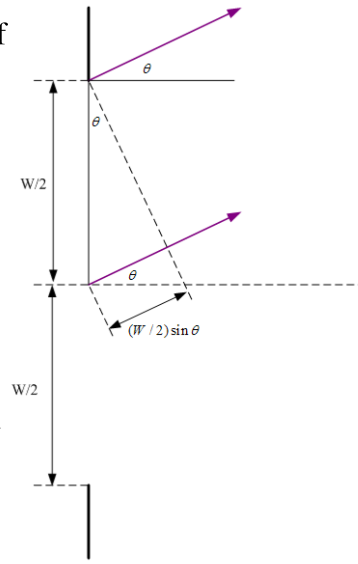


7. **Diffraction :** The bending of light around objects (into what would otherwise be a shadowed region) is known as diffraction. Diffraction occurs when light passes through very small apertures or near sharp edges. Diffraction is actually just interference, with the difference being only in the source of the interfering waves. Interference from a single slit, as in the figure here, is called diffraction.



We can get an interference pattern with a single slit provided its size is approximately equal to the wavelength of the light (neither too small nor too large).

8. Let's work out the condition necessary for diffraction of light from a single slit. With reference to the figure, imagine that a wave is incident from the left. It will cause secondary waves to be radiated from the edges of the slit. If one looks at angle θ, the extra distance that the wave emitted from the lower slit must travel is $W\sin\theta$. If this is a multiple of the wavelength λ, then constructive interference will occur. So the condition becomes $W\sin\theta = m\lambda$ with $m = \pm1, \pm2, \pm3...$ So, even from a single slit one will see a pattern of light and dark fringes when observed from the other side.



9. Light with wavelength of 511 nm forms a diffraction pattern after passing through a single slit of width 2.2×10^{-6}m. Find the angle associated with (a) the first and (b) the second bright fringe above the central bright fringe.

SOLUTION: For $m = 1$, $\theta = \sin^{-1}\left(\frac{m\lambda}{W}\right) = \sin^{-1}\left(\frac{(1)(511\times10^{-9}m)}{2.20\times10^{-6}m}\right) = 13.4^\circ$

For $m = 2$, $\theta = \sin^{-1}\left(\frac{m\lambda}{W}\right) = \sin^{-1}\left(\frac{(2)(511\times10^{-9}m)}{2.20\times10^{-6}m}\right) = 27.7^\circ$

10. Diffraction puts fundamental limits on the capacity of telescopes and microscopes to separate the objects being observed because light from the sides of a circular aperture interferes. One can calculate that the first dark fringe is at $\theta_{\min} = 1.22\frac{\lambda}{D}$, where D is diameter of the aperture. Two objects can be barely resolved if the diffraction maximum of one object lies in the diffraction minimum of the second object. Clearly, the larger D is, the smaller the angular diameter separation. We say that larger apertures lead to better resolution.

QUESTIONS AND EXERCISES

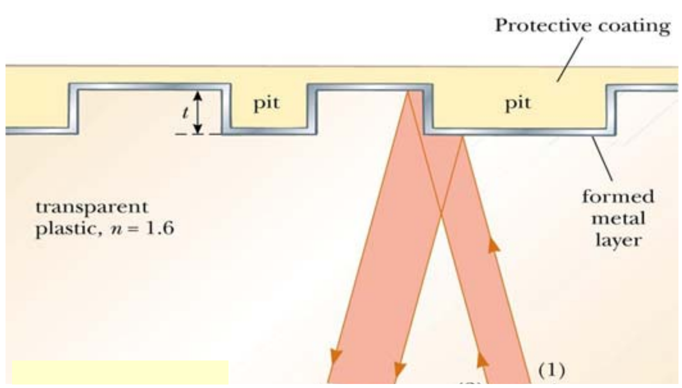
Q.1 Sound coming from two separate loudspeakers is readily heard to interfere. Why is not possible to demonstrate interference of waves coming from two separate light sources?

Q.2 In the two-slit experiment (see the diagram next to point no. 2 above) what would happen to the pattern on the third screen if one of the slits is covered up?

Q.3 Simplify $f(t) = \sin \omega t + \cos \omega t$ using a trigonometric identity that you certainly know. Then make a plot of $f^2(t)$ from $t = 0$ to $2\pi / T$.

Q.4 In a double slit experiment, green light of wavelength 550 nm illuminates slits that are 1.5mm apart. The screen is 2 m away. What will be the separation between the dark fringes?

Q.5 A compact disc player uses a laser to reflect light from the metal layer deposited on a protective coating, as shown below. When the light reflected from both segments is combined at the receiver, interference from the two waves results. We can therefore whether there is "one" or "zero" at that point on a CD.



Answer the following:

a) A CD has about 700 MB (megabytes) recorded upon it. Give a rough estimate of how much area is needed for one byte.

b) For a byte to be detected by interference using a green light laser, what should be t approximate depth t of the pit?