LUMS School of Science and Engineering

PHY-201 **Problem Set # 6 - Solutions** October 22, 2012

Problem 1

The wave function for the particle is given by:

$$\psi(x) = \sqrt{\frac{a}{\pi(x^2+a^2)}} \quad (a > 0 \text{ and } -\infty < x < +\infty) \ . \tag{1}$$

Let the event be that we somehow locate the particle between $x = -a$ and $x = +a$. We calculate the probability of this event:

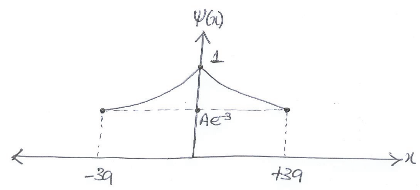
$$P = \int_{-a}^{+a} |\psi(x)|^2 dx = \int_{-a}^{+a} \frac{a}{\pi(x^2+a^2)} \, dx = \frac{2}{\pi}\int_0^a \frac{\frac{1}{a}\, dx}{(1+\frac{x^2}{a^2})} = \frac{1}{2} \ . \tag{2}$$

Problem 2

The particle wave function is:

$$\psi(x) = Ae^{-|x|/a} \ . \tag{3}$$

A sketch is shown below:



The value A can be found by requiring the state to be normalized:

$$1 = \int_{-\infty}^{+\infty} |\psi(x)|^2 dx = |A|^2 \int_{-\infty}^{+\infty} e^{-2|x|/a} \, dx = a|A|^2 \ \Rightarrow |A| = \frac{1}{\sqrt{a}} \ .$$

Indeed the simplest choice for A is:

$$A = \frac{1}{\sqrt{a}} \ . \tag{4}$$

Next, we calculate the probability that the particle is found between the positions $x = -a$ and $x = +a$.

$$P = \int_{-a}^{+a} |\psi(x)|^2 dx = \int_{-a}^{+a} \frac{1}{a} \, e^{-2|x|/a} \, dx = 1 - e^{-2} \approx 0.865 \ . \tag{5}$$

Problem 3

Consider the proton to be a particle in a box. Then the energy levels are given by the familiar expression:

$$E_n = \frac{n^2 h^2}{8mL^2} \ . \tag{6}$$

The photon energy is then:

$$E = E_2 - E_1 = \frac{3h^2}{8mL^2} \approx 6.16 \text{ MeV} \ , \tag{7}$$

and the wavelength is given by:

$$\lambda = \frac{hc}{E} \approx 2 \times 10^{-13} \text{ m} \ . \tag{8}$$

This wavelength belongs to the ultra violet region of the electro magnetic spectrum.

Problem 4

Note that probabilities are proportional to intensities, which are in turn proportional to the wave amplitudes. Consequently if we take the amplitude of the wave from slit 2 to be A, then that from slit 1 is 5A. Note that the ratio of the squares comes out to be 25: in accordance with the problem statement. Now at a maximum, the amplitudes add and the resultant amplitude is: $A_{max} = 5A + A = 6A$. Moreover at a minimum, there is incomplete destructive interference and the amplitude is non zero: $A_{min} = 5A - A = 4A$. The ratio of the probability of an electron arriving at the maximum to that of its arrival at the minimum, is simply the ratio of the respective amplitudes mod squared:

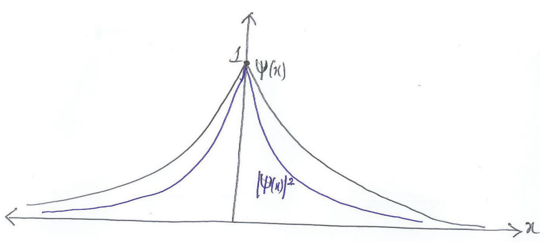
$$\frac{\text{Probability of arrival at maximum}}{\text{Probability of arrival at minimum}} = \frac{|A_{max}|^2}{|A_{min}|^2} = \frac{36}{16} = \frac{9}{4} \ . \tag{9}$$

Problem 5

The wave function for the electron is given to be:

$$\psi(x) = Ae^{-\alpha|x|} \quad (-\infty < x < +\infty) \ . \tag{10}$$

A sketch of this function along with that of the associated probability density is shown below: The wave function (apart from the mentioned draw back) seems to be quite reasonable. It



is a smooth function (except at $x = 0$) so that the derivatives and hence the Schrodinger's Equation is defined every where. Moreover it seems normalizable, in the sense that we can associate a unit probability of finding the particle somewhere in the universe. Lets find out whether this is true:

$$1 = \int_{-\infty}^{+\infty} |\psi(x)|^2 \, dx = |A|^2 \int_{-\infty}^{\infty} e^{-\alpha|x|} dx = \frac{2}{\alpha}|A|^2 \Rightarrow |A| = \sqrt{\frac{\alpha}{2}} \Rightarrow A = \sqrt{\frac{\alpha}{2}} \ . \tag{11}$$

Clearly the wave function comes out to be normalizable, and the probability of finding the electron in the region $-\frac{1}{2\alpha} < x < +\frac{1}{2\alpha}$ is:

$$P = \int_{-1/2\alpha}^{+1/2\alpha} |\psi(x)|^2 dx = \frac{\alpha}{2} \int_{-1/2\alpha}^{+1/2\alpha} e^{-2\alpha|x|} \, dx = \frac{1}{2}(1 - e^{-1}) \approx 0.316 \ . \tag{12}$$