LUMS School of Science and Engineering

PHY-201 **Assignment # 4 - Solutions** October 01, 2012

Problem 1

(a) The Compton shift in wavelength is given by the following formula:

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{mc}(1 - \cos\phi) \ . \tag{1}$$

Here m is the electron mass, ϕ the scattering angle, λ the incident photon wavelength, λ' the scattered photon wavelength, and $\Delta\lambda$ the required Compton Shift. Note that from the problem statement: $\phi = 37°$, and the right hand side of (1) can be evaluated. This gives the Compton Shift:

$$\Delta\lambda = 4.89 \times 10^{-13} \text{ m} \ . \tag{2}$$

(b) Consider the incident photon. It has energy $E = 3 \times 10^5$ eV $= 4.8 \times 10^{-14}$ J. From this we can calculate the wavelength:

$$\lambda = \frac{hc}{E} = 4.14 \times 10^{-12} \text{ m} \ . \tag{3}$$

This allows the calculation of the scattered wavelength:

$$\lambda' = \lambda + \Delta\lambda = 4.63 \times 10^{-12} \text{ m} \ . \tag{4}$$

Finally, this wavelength gives the energy of the scattered photon:

$$E' = \frac{hc}{\lambda'} = 4.29 \times 10^{-14} \text{ J} = 268.3 \text{ keV} \ , \tag{5}$$

the required answer.

(c) We can label the lorentz factor for the moving electron by γ, and write the equation of energy conservation:

$$E + mc^2 = E' + \gamma mc^2 \Rightarrow \gamma mc^2 = E - E' + mc^2 = 543 \text{ keV} \ , \tag{6}$$

the required answer.

Problem 2

The mass of the student is $m = 80$ kg, and the door width is $w = 0.75$ m. In the process of passing through the door, the student has an associated De-Broglie wavelength λ, and the door acts as a slit of width w. Thus the student expects to experience diffraction. However as in the problem statement, the laid out criterion for noticable diffraction is:

$$w < 10\lambda \Rightarrow \lambda > \frac{w}{10} \ . \tag{7}$$

Lets examine the requirement imposed by (7) on the speed:

$$\lambda = \frac{h}{mv} > \frac{w}{10} \ \Rightarrow \ v < \frac{10h}{mw} \approx 1.1 \times 10^{-34} \text{ m/s} \ . \tag{8}$$

So in order to experience diffraction, the maximum speed that the student can have is given by (8) above. Taking one foot step to be about 1 m long and using the speed found above, it takes the student a minimum of 9.1×10^{33} s $\approx 2.9 \times 10^{26}$ y to cross the door. That not only exceeds the human lifespan , but also that of the universe itself.

Problem 3

Consult the diagram in the problem statement, and use the quantities as mentioned. At the vicinity of the first electron, the Compton Scattering Formula reads:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \ . \tag{9}$$

A straight forward use of basic trigonometry shows the scattering angle at the vicinity of the second electron to be $\pi - \theta$, so that the scattering formula becomes:

$$\lambda'' - \lambda' = \frac{h}{mc}[1 - \cos(\pi - \theta)] \ . \tag{10}$$

Adding (9) and (10) and simplifying we get:

$$\lambda'' - \lambda = \frac{2h}{mc} \ . \tag{11}$$

This is the required answer. But the interesting thing to note is that (11) can be recast into the form:

$$\lambda'' - \lambda = \frac{h}{mc}(1 - \cos\pi) \ , \tag{12}$$

where π is the scattering angle of the third photon in relation to the first. And the scattering formula somehow seems transitive, which it indeed is.

Problem 4

(a) We are given an electron with energy 20 GeV $= 3.2 \times 10^{-9}$ J (Kinetic). The requirement here is to work out the lorentz factor γ. This can be done as in the following:

$$K = (\gamma - 1)mc^2 \Rightarrow \gamma = 1 + \frac{K}{mc^2} \approx 39082.24 \ . \tag{13}$$

(b) It is tempting to use the lorentz factor found in (13) to find the electron speed v, and then to find the momentum by using $p = \gamma mv$. However this approach is a bit tedious and not quite fruitful for subsequent calculations in the problem (try it out and feel convinced). An easier detour is to do the following:

$$E^2 = (mc^2)^2 + (pc)^2 \Rightarrow (K + mc^2)^2 = (mc^2)^2 + (pc)^2 \Rightarrow p = \frac{1}{c}\sqrt{K(K + 2mc^2)}$$

$$= 1.067 \times 10^{-17} \text{ kg.m/s} \ . \tag{14}$$

(c) The De-Broglie wavelength is simply given by:

$$\lambda = \frac{h}{p} = 6.21 \times 10^{-17} \text{ m} \ . \tag{15}$$

(d) Compared to an atomic diameter of $D = 10^{-14}$ m, the electron wavelength is considerably small:

$$\lambda \approx 0.0062D \ . \tag{16}$$

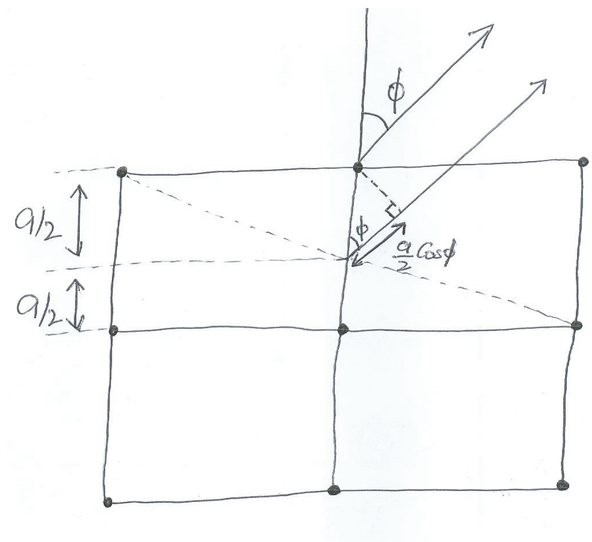
This completes the solution.

Problem 5

The kinetic energy of the electrons is given to be $K = 54$ eV $= 8.64 \times 10^{-18}$ J. The electron momentum can be found as in (14), and then utilized to calculate the De-Broglie wavelength as in (15):

$$\lambda = 1.67 \times 10^{-10} \text{ m} \ . \tag{17}$$

As shown in the problem statement diagram, electrons bounce off the indicated adjacent oblique planes and exhibit interference. The following diagram helps to explain the fact that the path difference between the reflected waves is $\frac{a}{2} + \frac{a}{2}\cos\phi$:



Then clearly the condition for constructive interference is:

$$\frac{a}{2} + \frac{a}{2}\cos\phi = m\lambda \ , \tag{18}$$

and the first minimum corresponds to $m = 1$. Solving for a while choosing $m = 1$ above we get:

$$a = \frac{2\lambda}{1 + \cos\phi} = 2.03 \times 10^{-10} \text{ m} \ . \tag{19}$$

Problem 6

(a) Label the the relevant direction along the pond as x. Then the position uncertainty is $\Delta x = 1$ m. The mass of the duck is $m = 2$ kg, and the value of the Planck's constant is $h = 2\pi$ J.s. The uncertainty principle reads:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \Rightarrow \Delta p_x = m\Delta v_x \geq \frac{\hbar}{2\Delta x} \Rightarrow \Delta v_x \geq \frac{\hbar}{2m\Delta x}$$

$$\Rightarrow \Delta v_{x_{min}} = \frac{\hbar}{2m\Delta x} = \frac{h}{4\pi m \Delta x} = 0.25 \text{ m/s} . \tag{20}$$

(b) Lets demarcate the pond: we take one end (the left end) to be located at $x = 0$ m, and the other end (the right end) to be located at $x = 1$ m. Now what can happen at most is that the duck is located at the left end and has a leftward speed as in (20), or it could be located at the right end and have a rightward speed as in (20). In either case, the duck could have traveled a distance of $(0.25)(5) = 1.25$ m away from the end of the pond. Thus the total position uncertainty at the end of 5 s would be:

$$1.25 \text{ m} + 1.00 \text{ m} + 1.25 \text{ m} = 3.5 \text{ m} . \tag{21}$$

This is the required answer.

Problem 7

Suppose the bullet is fired in the xy plane from the origin, along the $+x$ axis. Its mass is $m = 0.001$ kg, and the speed along the x axis is $v_x = 100$ m/s. Now the bullet is fired from a barrel which has a diameter of $d_0 = 0.002$ m. You can take this barrel to be oriented such that the x axis is an axis of symmetry. Thus the extreme y coordinates of the barrel are $y = +0.001$ m and $y = -0.001$ m, and clearly the y position uncertainty of the bullet (at the instant of firing) is $\Delta y = 0.002$ m. We can use the uncertainty principle to work out the minimum uncertainty in its speed the y direction:

$$\Delta y \Delta p_y = m\Delta y \Delta v_y \geq \frac{\hbar}{2} = \frac{h}{4\pi} \Rightarrow \Delta v_y \geq \frac{h}{4\pi m \Delta y}$$

$$\Rightarrow \Delta v_{y_{min}} = \frac{h}{4\pi m \Delta y} . \tag{22}$$

Now the screen (target) is located farther down the $+x$ axis, and lies parallel to the yz plane. The minimum speed uncertainty found above will certainly result in a minimum y position uncertainty at the vicinity of the target. We label this as $\Delta y'_{min}$ and the travel time as Δt. Then clearly:

$$\Delta y'_{min} = d_0 + 2\Delta v_{y_{min}} \Delta t = d_0 + \frac{h}{2\pi m \Delta y} \Delta t , \tag{23}$$

where we have utilized (22). Note the factor of 2 in the center equality above. It appears because the speed can be directed either way at either end of the barrel, that is along the $+y$ direction or the $-y$ direction. Now at the vicinity of the target, we are assured a y position uncertainty as found in (23). It is this assured uncertainty that is measured at the target as a circle of diameter 0.01 m. Thus we can set $\Delta y'_{min} = 0.01$ m in (23) and solve for Δt:

$$\Delta t = \frac{2\pi m \Delta y}{h} \, (\Delta y'_{min} - d_0) \approx 4.8 \times 10^{18} \text{ y} , \tag{24}$$

a quantity greater than the current age of the universe. This means that particle never gets to the slit: A contradiction. Consequently the situation described in the problem is not possible.