LUMS School of Science and Engineering

PHY-201 **Assignment # 3 - Solutions** September 24, 2012

Problem 1

In order to obtain an interference pattern for waves from two independent sources, it is absolutely necessary that the two sources emit waves which are of the same frequency, and are precisely in phase. Although this can be achieved for sound waves, it is not possible for light waves. This is precisely the reason for using a single source of light in the Young's double slit experiment.

Problem 2

Assuming both slits to be point like (as in the Double Slit Experiment), covering up one of them would cause the alternating dark and bright fringes to disappear and the whole screen to turn bright.

Problem 3

The function provided here is a sum of two sinusoids:

$$f(t) = \sin \omega t + \cos \omega t \ . \tag{1}$$

Of course any thing simpler is a single sinusoid. So lets combine the Sine and the Cosine into a single Cosine. And for this we invoke the following mathematical identity:

$$A \cos(\omega t - \alpha) = A \sin \alpha \sin \omega t + A \cos \alpha \cos \omega t \ . \tag{2}$$

We desire to find an A and α such that $f(t)$ can be written as $A \cos(\omega t - \alpha)$. So we equate the right hand sides of the above two equations and solve for A and α, which yields $\alpha = \frac{\pi}{4}$ and $A = \sqrt{2}$. Thus:

$$f(t) = \sqrt{2} \cos(\omega t - \frac{\pi}{4}) \ . \tag{3}$$

Squaring gives:

$$f^2(t) = 2 \cos^2(\omega t - \frac{\pi}{4}) \ , \tag{4}$$

However, if we choose to square and then simplify, we get a form that is more conveniently sketched:

$$f^2(t) = (\sin \omega t + \cos \omega t)^2 = \sin^2 \omega t + \cos^2 \omega t + 2 \sin \omega t \cos \omega t = 1 + \sin 2\omega t \ . \tag{5}$$

A hand sketch is shown in Figure 1. As for the function $f(t) = \sin \omega t + \frac{1}{2} \cos \omega t$, the simplification process is quite tedious and even the simplest form does not give a function which can be readily sketched as done above. Consequently, a computerized plot has been included Figure 2.

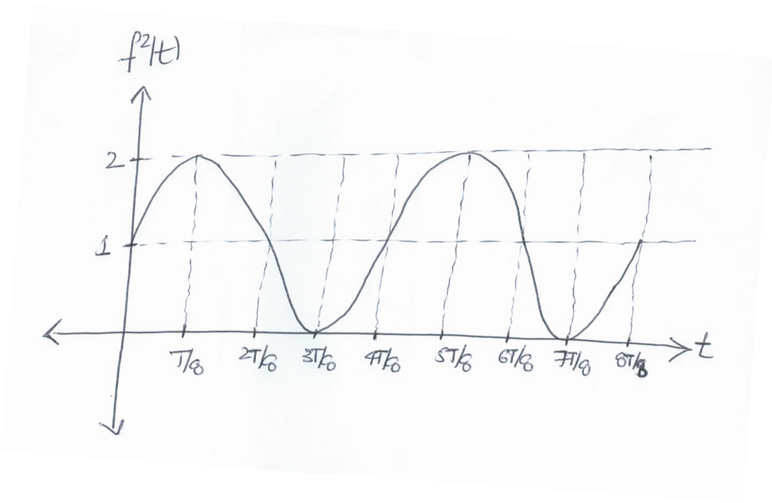

Figure 1: Graph of $f^2(t) = (\sin \omega t + \cos \omega t)^2$

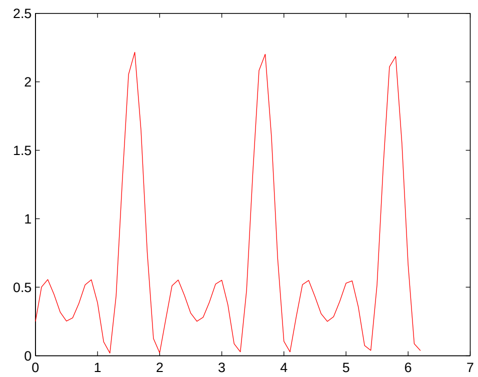

Figure 2: Graph of $f^2(t) = (\sin \omega t + \frac{1}{2} \cos \omega t)^2$

Problem 4

Shown in Figure 3 is the usual schematic diagram of the Double Slit experiment. We are given $\lambda = 550$ nm, $d = 1.5$ mm and $R = 2$ m. The task is to work out the inter-fringe spacing for dark fringes. Note that the ratio $\frac{d}{R}$ is exceptionally small, small enough so that the two light rays and the dotted line between them can essentially be treated as parallel. Then the condition for destructive interference is:

$$d \sin \theta_m = (m - \frac{1}{2})\lambda \qquad (m \text{ is an integer}) \;, \tag{6}$$

where θ_m is the angular displacement of the mth dark fringe. And in the limit of small angles we can make the approximation $\sin \theta_m \approx \tan \theta_m = y_m / R$ (y_m being the vertical displacement of the mth dark fringe from the center line), in the above equation. This gives:

$$\frac{d y_m}{R} \approx (m + \frac{1}{2})\lambda \;\;\Rightarrow\;\; y_m \approx (m + \frac{1}{2})\frac{R\lambda}{d} \;. \tag{7}$$

Moreover if we replace m with $m+1$ above, we get the vertical displacement of the $(m+1)$th

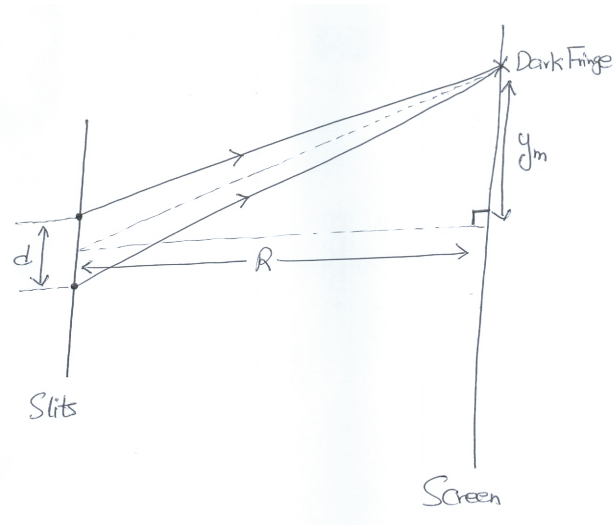

Figure 3: Schematic diagram of the Double Slit Experiment

dark fringe from the center line:

$$y_{m+1} = (m + \frac{1}{2})\frac{R\lambda}{d} \ . \tag{8}$$

Subtracting the above two results gives the inter-fringe spacing:

$$\Delta y = y_{m+1} - y_m = \frac{R\lambda}{d} \ . \tag{9}$$

Note how the inter-fringe spacing comes out to be independent of m. Now plugging in all the knowns (i.e. R, λ and d) gives us the result:

$$\Delta y = 0.73 \text{ mm} \ . \tag{10}$$

Problem 5

(a) Although smaller sizes are available, the average outer diameter of a CD is about $D_o = 120$ mm and that of the center spindle hole is about $D_i = 15$ mm. The area enclosed is given by:

$$A = \frac{\pi D_o^2}{4} - \frac{\pi D_i^2}{4} = 11137.5 \text{ mm}^2 \ . \tag{11}$$

We assume that all this area is utilized for recording about 700 MB. Then clearly, the area required per byte should be approximately:

$$\text{Byte Area} = \frac{11137.5 \text{ mm}^2}{700 \times 10^6 \text{ Bytes}} = 1.59 \times 10^{-5} \text{ mm}^2/\text{Byte} \ . \tag{12}$$

(b) Take typical green light wavelength to be $\lambda_0 = 550$ nm. Note that we used the symbol λ_0 to indicate the fact that this is the wavelength in vacuum. The wavelength in the plastic can be labeled λ. Let n denote the index of refraction of the plastic, then from the problem statement: $n = 1.6$. Now as shown in the figure, we are assuming nearly normal incidence. Hence the path difference between the two rays should be about twice the thickness: $2t$. Moreover, in order to know that a pit has arrived, destructive interference must occur between the rays. The condition is then:

$$2t = (m - \frac{1}{2})\lambda = (m - \frac{1}{2})\frac{\lambda_0}{n} \Rightarrow t = \frac{(m - \frac{1}{2})\lambda_0}{2n} \ . \tag{13}$$

The minimum thickness is given by choosing $m = 1$ above:

$$t_{min} = \frac{\lambda_0}{4n} = \frac{550 \text{ nm}}{(4)(1.6)} = 85.94 \text{ nm} \ . \tag{14}$$

Problem 6

Label the mirrored plate as plate M and the darkened plate as plate D, and let the momentum of each incident photon be p. Note that plate D absorbs all photons where as plate M reflects back each photon (along its path of incidence). Next, consider plate D placed at the origin in the $y - z$ plane and a photon at a point on the $+x$ axis. Let the photon be traveling in the $-x$ direction so that it is normally incident on plate D. We intend on using momentum conservation for the plate-photon system. Clearly for this system, the momentum before collision is $-p\hat{i}$. And this indeed is also the case after collision. However during the collision process, the photon is absorbed by the plate and consequently becomes non existent. Then it must be that after collision, plate D has momentum $-p\hat{i}$. Thus the momentum imparted by a photon to plate D is $-p\hat{i}$. Now consider repeating the analysis, but with plate D replaced with plate M. The momentum before collision is $-p\hat{i}$ (as before). But now during the collision the photon is reflected, meaning that it has momentum $+p\hat{i}$. And by momentum conservation, it must be that plate M has momentum $-2p\hat{i}$. Thus each photon imparts a momentum $-2p\hat{i}$ to plate M. Now the number of photons arriving per unit time on both plates can be taken to be the same: $\frac{\Delta N}{\Delta T}$, and so can the area of the two plates: A. And by Newton's Second Law, the force experienced by each plate is its rate of change of momentum. Thus for plate D:

$$\vec{F}_D = -pA\frac{\Delta N}{\Delta t}\hat{i} \ , \tag{15}$$

and for plate M:

$$\vec{F}_M = -2pA\frac{\Delta N}{\Delta t}\hat{i} \ . \tag{16}$$

That is, M experiences a force twice as great as D. This force, divided by the plate area, is something that is called the Radiation Pressure. Note that the Radiation Pressure is twice as large for a reflecting plate than for an absorbing plate.

Problem 7

The photon must provide a minimum energy of 2.2 eV to the electron. Since the energy of a photon is given by $E = hf$, the minimum energy indeed corresponds to the minimum frequency, which can be worked out as in the following.

$$E_{min} = 2.2 \text{ eV} = (2.2)(1.6 \times 10^{-19}) \text{ J} = 3.52 \times 10^{-19} \text{ J} \ .$$

Also:

$$E_{min} = hf_{min} = (6.626 \times 10^{-34})f_{min} \ .$$

Using the right hand sides of the above two equations gives the desired answer:

$$f_{min} = 5.31 \times 10^{14} \text{ Hz}. \tag{17}$$

A frequency larger than the one calculated above, means an energy in excess of E_{min}. Clearly, E_{min} would be utilized in ejecting the electron, and the excess would manifest itself as the electron kinetic energy.

Problem 8

The power of the laser pulse is simply the energy carried per unit time:

$$P = \frac{\Delta E}{\Delta t} = \frac{2}{0.001} = 2000 \text{ W} . \quad (18)$$

For green light (given wavelength $\lambda = 550$ nm), the energy of one single photon is:

$$E_{ph} = \frac{hc}{\lambda} = \frac{(60626 \times 10^{-34})(2.998 \times 10^{8})}{550 \times 10^{-9}} = 3.61 \times 10^{-19} \text{ J} . \quad (19)$$

Consequently, the number of photons carried by the wave per unit time should be:

$$n = \frac{P}{E_{ph}} = 5.5 \times 10^{21} \text{ photons/s} . \quad (20)$$

Similarly for blue light, we can take the characteristic wavelength to be $\lambda = 434$ nm, and repeat the above procedure to get

$$n = 4.4 \times 10^{21} \text{ photons/s} . \quad (21)$$