LUMS School of Science and Engineering

PHY-201 **Assignment # 2 - Solutions** September 14, 2012

Problem 1

Take the frame of the galaxy center to be S: our usual inertial frame. Take one jet to be traveling with speed $0.75c$ along positive x axis (lets call this jet 1) and the other to be traveling with speed $0.75c$ along the negative x axis (call this jet 2). Take the rest frame of jet 1 to be in standard configuration with S and label it S'. Let the velocity parameter be v. Then clearly:

$$v = 0.75c \; . \tag{1}$$

Now the velocity of jet 2 in S is:

$$u_x = -0.75c \; . \tag{2}$$

We can calculate its velocity in the rest frame of jet 1 (i.e. S') by the usual velocity transformation formula:

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2} \; . \tag{3}$$

Using (1) and (2) in (3) gives:

$$u'_x = -0.96c \text{ (the speed of jet 2 in the rest frame of jet 1)} \; . \tag{4}$$

We can now invoke the relativity principle and claim the following:

Each jet ascribes the same speed to the other. If jet 1 observes jet 2 moving away with speed 0.96c, then jet 2 also observes jet 1 to be moving away with speed 0.96c. Thus we have calculated the speed of each jet relative to the other.

Problem 2

(a) Here we are given two inertial frames S and S' in standard configuration, with velocity parameter v. Two events occur: Event 1, the flashing of the red light, and Event 2, the flashing of the blue light. The coordinates in S are given to be:

$$x_1 = 3\text{m} \; , \; t_1 = 1 \times 10^{-9}\text{s} \; , \; x_2 = 5\text{m} \; , \; t_2 = 9 \times 10^{-9}\text{s}. \tag{5}$$

It is known that the two events occur at the same place in S' (i.e. $\Delta x' = 0$), and we are to calculate the velocity parameter v. Then:

$$\Delta x' = \gamma(\Delta x - v\Delta t) = 0 \Rightarrow v = \frac{\Delta x}{\Delta t} = \frac{(5-3)}{(9 \times 10^{-9}) - (1 \times 10^{-9})} = 0.8339c \; . \tag{6}$$

(b) Next, we calculate the location of the two events in S'. Clearly:

$$x'_1 = \gamma(x_1 - vt_1) = \frac{3 - (0.8339c)(1 \times 10^{-9})}{\sqrt{1 - 0.8339^2}} = 4.9825\text{m} = x'_2 \; . \tag{7}$$

(c) Lastly, we compute t'_1. This can be done straight away from the Lorentz Transformation:

$$t'_1 = \frac{t_1 - vx_1/c^2}{\sqrt{1 - v^2/c^2}} = \frac{(1 \times 10^{-9}) - (0.8339c)(3)/c^2}{\sqrt{1 - 0.8339^2}} = -13.31\text{ns} \; . \tag{8}$$

Problem 3

Denote the particle mass to be m. In the first instance, the particle is accelerated from a speed of $0.2c$ to a speed of $0.3c$. The work that must be done in order to achieve this is the change in the particle's relativistic kinetic energy:

$$W = \Delta K = K_2 - K_1 = (\gamma_2 - 1)mc^2 - (\gamma_1 - 1)mc^2 = \left(\frac{1}{\sqrt{1-0.3^2}} - \frac{1}{\sqrt{1-0.2^2}} \right) mc^2$$

$$= (2.49 \times 10^{15})\ m\ . \tag{9}$$

Likewise, when the same particle is accelerated from a speed of $0.8c$ to a speed of $0.9c$, a similar calculation yields:

$$W = \Delta K = K_2 - K_1 = (\gamma_2 - 1)mc^2 - (\gamma_1 - 1)mc^2 = \left(\frac{1}{\sqrt{1-0.9^2}} - \frac{1}{\sqrt{1-0.8^2}} \right) mc^2$$

$$= (5.64 \times 10^{16})\ m\ . \tag{10}$$

Clearly the results derived here differ significantly, that is by orders of magnitude. The reason is that as and when the speed of a particle increases, so does its relativistic mass. And consequently it gets harder and harder (more and more work input is required) to produce the same change in the particle's velocity (which in this case is $0.1c$).

Problem 4

Here we have two space craft traveling relative to the Earth frame S: an enemy craft traveling at a speed of $0.8c$ and a pursuit craft traveling at speed $0.9c$. We need to calculate the speed of the enemy craft as measured by observers on the pursuit craft. So lets attach a reference frame to the pursuit craft and label this S'. Then it is clearly always possible to choose coordinates so that S and S' are in standard configuration. Moreover the velocity parameter between S and S' is $v = 0.9c$. The speed of the enemy craft in S is $u_x = 0.8c$. And we desire to calculate its speed in S'. This is achieved by writing out the velocity transformation equation and plugging in the knowns:

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2} = \frac{(0.8-0.9)c}{1-(0.9)(0.8)} = -0.357c\ . \tag{11}$$

Thus as viewed by observers on the pursuit craft, the enemy ship seems to be moving backwards with speed $0.357c$, which is precisely the speed of overtaking.

Problem 5

Consider the fusion process to be occuring in an inertial frame S. Take the particle with mass $m = 1.99 \times 10^{-26}$ kg to be moving along the positive x axis with speed $u = 0.5c$ and lorentz factor γ. Towards the right of this particle on the x axis, consider the particle with mass $m/3$ to be moving along the negative x direction with the same speed and lorentz factor. Let the primes denote the respective quantities after fusion and write out the equations of momentum and energy conservation:

$$\gamma mu - \gamma \frac{m}{3} u = \gamma' m' v'\ , \tag{12}$$

$$\gamma mc^2 + \gamma \frac{m}{3} c^2 = \gamma' m' c^2 \ . \tag{13}$$

It clearly follows from (13) that:

$$\gamma' m' = \frac{4}{3} \gamma m \ .$$

Using this result in (12) we can show that:

$$v' = \frac{u}{2} = 0.25c \ \Rightarrow \ \gamma' = 1.0328 \ . \tag{14}$$

Finally, using the information in (14) back in (12) we get the mass of the composite particle:

$$m' = 2.97 \times 10^{-26} \text{ kg}. \tag{15}$$

This completes the solution.

Problem 6

(a) Since an amount of energy $E = 2.86 \times 10^5$ J is released in the process, it must be that the mass of the product is smaller than the mass of the reactants. It is in fact the energy equivalent of this mass difference that is released.

(b) By the argument in (a), the mass difference is $\Delta m = E/c^2 = 3.18 \times 10^{-12}$ kg.

(c) The mass found above, is an exceedingly small fraction of 9 g. Thus detection seems unlikely.

Problem 7

The fact that the plant operates at 80% of its full capacity, means that the operating power is $P = 0.8 \times 1 \times 10^9$ W $= 8 \times 10^8$ W. The plant is kept operational at this power for 3 years, which means that an amount of energy $E = (8 \times 10^8)(3)(365)(24)(3600) = 7.57 \times 10^{16}$ J is liberated. The mass equivalent of this energy is precisely the fuel mass that is consumed to achieve this: $\Delta m = E/c^2 = 0.842$ kg.

Problem 4 (Assignment 1)

The characterizing wavelengths for red and green light are respectively: $\lambda_R = 650$ nm and $\lambda_G = 520$ nm. Note that in the rest frame of the observer with a receding source, the Doppler's Shift Formula reads:

$$f_O = \sqrt{\frac{c-v}{c+v}} f_S \ , \tag{16}$$

where f_O is the observed frequency, f_S is the source frequency (proper) and v is the speed of the source, directed away from the observer. If instead, we have the source moving toward the observer, a simple replacement of $v = -|v|$ above gives the correct equation:

$$f_O = \sqrt{\frac{c+|v|}{c-|v|}} f_S \ . \tag{17}$$

Let us (for the moment) choose to work in the observer's rest frame. Consequently, the result that concerns us is (17). Expressing frequencies in terms of wave lengths and solving for the speed, we get:

$$|v| = \frac{(\frac{\lambda_S}{\lambda_O})^2 - 1}{(\frac{\lambda_S}{\lambda_O})^2 + 1}\ c\ . \tag{18}$$

We can now require the source wavelength to be that of red light and the observed wavelength to be that of green light, by choosing $\lambda_S = \lambda_R = 650$ nm and $\lambda_O = \lambda_G = 520$ nm in (18), so that:

$$|v| = 0.22c\ . \tag{19}$$

Thus as measured by the observer, the source moves towards him at the speed found above. And by symmetry, it must be that in the rest frame of the source, the observer drives towards it at this same speed. This is the required answer.