LUMS School of Science and Engineering

PHY-201 **Assignment # 1 - Solutions** September 07, 2012

Problem 1

(a) Since the case of a stationary meter stick is trivial, we will assume the meter stick to be moving in an Inertial Reference frame S. The key idea in measuring the length of this stick is to ensure that its end points are observed simultaneously. The coordinates of both the end points should be read off at the same time (i.e. simultaneously), so that the length can be calculated from the usual distance formula. Note that observing the ends at different instances of time would give the incorrect result. Now having carefully followed the aforementioned procedure, we can claim to have made the correct length measurement in S. The point worth noting is this:

In order to make the correct length measurement in S, we required the time interval between measurements to be zero.

(b) Quite analogously, we could be required to make the correct time measurement in S. In this case we need to be cautious of our spatial location. Suppose we utilize a stop watch for this process. Then we need to ensure that we, and more importantly our stop watch, do not undergo a change of space coordinates during the measurement process. So with the watch fixed at a spatial location, we can measure a time interval and claim to have made a correct time measurement in S. Note that in case the clock moves during the measurement process, our claim is no longer valid because of time dilation: the moving clock slows down and displays a smaller time in comparison to that which has elapsed in S. The idea worth noting is this:

In order to make the correct time measurement in S, we required the spatial interval between measurements to be zero. The measurements are simultaneous in the sense of space coordinates.

Problem 2

Take the earth frame to be S and that of the space ship to be S'. Moreover, take S and S' to be in standard configuration with velocity parameter v. Then these two inertial frames are connected by the usual Lorentz Transformation, with $v = 0.5c$. The missile speeds in S' are $u'_x = \pm 0.6c$, with the plus sign signifying the fact that the missile is fired away from earth and the minus sign doing the opposite. We are looking out for the missile speeds in S, that is u_x. The velocity transformation formula reads:

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2} \ . \tag{1}$$

Using $v = 0.5c$ and $u'_x = +0.6c$ in (1) gives the missile speed in the earth frame:

$$u_x = +0.846c \text{ (away from earth)} \ .$$

Simlarly, using $v = 0.5c$ and $u'_x = -0.6c$ in (1) yields:

$$u_x = -0.143c \text{ (toward earth)} \ .$$

This completes the solution.

Problem 3

The clock aboard the airplane (assumed to be an inertial frame) travels with a speed (assumed constant) relative to the earth (also assumed to be an inertial frame). The moving clock should slow down in accordance with the following time dilation formula:

$$\Delta t = \gamma \Delta \tau \ , \tag{2}$$

where $\Delta\tau$ is the time elapsed on the moving clock aboard the airplane, Δt is the corresponding time elapsed on an earth clock, and $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$. Since $\gamma > 1$, the moving clock is observed to go slow by a factor of γ. Let us calculate this factor:

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = (1-\frac{v^2}{c^2})^{-1/2} = 1 + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots \approx 1 + \frac{v^2}{2c^2} \ (\because v = 1000 \text{ km/h} << c)$$

$$\Rightarrow \gamma \approx 1 + 4.29 \times 10^{-13} \ .$$

Dropping the Δ notation in (2) and solving for t in terms of τ allows us to write the following:

$$\tau = \frac{1}{\gamma}\ t \approx (1 - \frac{v^2}{2c^2})\ t \Rightarrow t - \tau = \frac{v^2}{2c^2}\ t = (4.29 \times 10^{-13})\ t \ . \tag{3}$$

So if according to earth time the airplane continues to fly for a week, we can find the clock difference from (3) to be about 260 ns.

Problem 4

Solutions to be provided later.

Problem 5

To an observer in S, the clocks in S' will not appear synchronized. In the following we will find out as to why this should be so. Consider yourself to be located at rest relative to S. Clearly the clocks in S would appear synchronized. Now consider viewing the clocks in S'. Pick any two adjacent clocks, and label the left clock as clock 1 and the right one as clock 2. Define event 1 to be the event at which clock 1 strikes noon. Similarly define event 2 to be the event at which clock 2 strikes noon. Let the separation between the two clocks in S' be d, and consider the following difference version of the lorentz transformation:

$$\Delta t = \gamma(\Delta t' + \frac{v\Delta x'}{c^2}) \ . \tag{4}$$

In the equation above, we can identify that $\Delta x' = d$ (since this is the separation between the two clocks in S'), and $\Delta t' = 0$ (since the clocks are synchronized in S'). Using this information in (4) we get:

$$\Delta t = \frac{\gamma v d}{c^2} > 0 \ .$$

The positive sign means that as viewed in S, event 2 has a larger time coordinate than event 1. That means clock 2 strikes 12 after clock 1, and it must be that clock 2 lags behind clock 1. Thus as viewed in S, for any pair of adjacent clocks, the right clock lags behind the left one by an amount of time $\frac{\gamma v d}{c^2}$. Moreover, this lag is the same for any pair of adjacent clocks. This is very accurately depicted in the figure shown in the problem statement. So to an observer in S, the clocks of S' do not appear synchronized.

Problem 6

In the reference frame in which the Earth and planet X are at rest, the respective speeds of Speedo and Goslo are $v_s = 0.95c$ and $v_g = 0.75c$, and the respective travel times come out to be:

$$\Delta t_s = \frac{20\text{ly}}{0.95c} = \frac{20}{0.95} = 21.05\text{y, and } \Delta t_g = \frac{20\text{ly}}{0.75c} = \frac{20}{0.75} = 26.67\text{y} \; . \tag{5}$$

However, each of Speedo and Goslo experience time dilation because of their motion. The result is that their biological clocks slow down, and the faster sibling ages less during travel. Denote the time experienced by Speedo to be $\Delta\tau_s$, then this quantity can be calculated from the time dilation formula as follows:

$$\Delta\tau_s = \frac{\Delta t_s}{\gamma_s} = \Delta t_s\sqrt{1 - v_s^2/c^2} = 21.05\sqrt{1 - 0.95^2} = 6.57\text{y} \; . \tag{6}$$

Likewise the time experienced by Goslo can be shown to be:

$$\Delta\tau_g = \frac{\Delta t_g}{\gamma_g} = \Delta t_g\sqrt{1 - v_g^2/c^2} = 26.67\sqrt{1 - 0.75^2} = 17.64\text{y} \; . \tag{7}$$

So Speedo takes 6.57 y to get to planet X, and as can be seen from (5), he has to wait another $26.67 - 21.05 = 5.62$ y on planet X before Goslo arrives there. So by the time Goslo gets there, Speedo has aged by $6.57 + 5.62 = 12.19$ y. In the mean time Goslo has aged by 17.64 y (his travel time from Earth to planet X). Thus at the instant he gets to planet X, Goslo is older than Speedo by $17.64 - 12.19 = 5.45$ y.

Problem 7

In the figure shown in the problem statement, take the $+x$ axis to be along the direction of motion and the $+y$ axis to be directed upward. It is easy to realize that the x component the rod's length undergoes lorentz contraction because of motion, where as the y component does not. So if one was in the rest frame of the rod (the proper frame), the y component should appear the same and the x component some what larger (in accordance with length contraction):

$$L_{oy} \text{ (the } y \text{ component in the rest frame)} =$$

$$L_y \text{ (the } y \text{ component in the frame in which the rod moves)} = 2\sin 30^\circ = 1 \text{ m.} \tag{8}$$

And:

$$L_{ox} = \gamma L_x = \frac{L_x}{\sqrt{1 - v^2/c^2}} = \frac{2\cos 30^\circ}{\sqrt{1 - 0.995^2}} = 17.34 \text{ m.} \tag{9}$$

Thus the proper length of the rod is: $L_o = \sqrt{L_{0x}^2 + L_{0y}^2} = \sqrt{17.34^2 + 1^2} = 17.37$ m, and the angle of orientation can be found as $\theta_o = \arctan\frac{L_{oy}}{L_{ox}} = 3.3^\circ$.