LUMS School of Science and Engineering

PHY201 Modern Physics – Assignment # 5

Q.1 Suppose that $|1\rangle, |2\rangle, |3\rangle$ are any three orthonormal states, meaning $\langle i | j \rangle = \delta_{ij}$, where $\delta_{ij} = 1$ if i = j and is zero otherwise (called Kronecker delta).

Suppose, for some reason, 10^6 identical systems have been prepared so that each system is in exactly the same state $|\psi\rangle$. Further suppose that,

$$|\psi\rangle = \sqrt{\frac{1}{5}}|1\rangle - i\sqrt{\frac{1}{5}}|2\rangle - \sqrt{\frac{3}{5}}|3\rangle$$

a) According to the rules of quantum mechanics, about how many systems will be found to be in each of the states 1, 2, 3? Why is your answer not exact?

b) How will you interpret $\langle i|\psi\rangle$ for each value of i $(i = 1, 2, 3)$?

c) What is the interpreation of $\langle\psi|i\rangle$?

d) Let's define the unit operator $\hat{I}$ such that $\hat{I}|i\rangle = |i\rangle$. Also define an operator $\hat{C}$ such that

$$\hat{C}|1\rangle = |2\rangle, \hat{C}|2\rangle = |3\rangle, \quad \hat{C}|3\rangle = |1\rangle.$$

Describe in words what this operator is doing.

e) Find $|\varphi\rangle = \hat{C}|\psi\rangle$ and then compute both $\langle\varphi|\psi\rangle$ and $\langle\psi|\varphi\rangle$. Interpret your result.

f) Define a new operator made from $\hat{I}$ and $\hat{C}$, $\hat{D} = \sqrt{\frac{2}{3}}\,\hat{I} + i\sqrt{\frac{1}{3}}\,\hat{C}$. What is the effect of operating with $\hat{D}$ on each of the states $|j\rangle$, i.e. find $\hat{D}|j\rangle$ (which can also be equally well written as $|\hat{D}j\rangle$).

g) For i, j with i and j both running over 1, 2, 3 compute the "matrix elements" of $\hat{D}$, i.e. the set of 9 numbers $D_{ij} = \langle i|\hat{D}|j\rangle$ and arrange them as a matrix. Is it true that $D_{ji} = D_{ij}^*$?

h) Define $\hat{C}^2 = \hat{C}\hat{C}, \quad \hat{C}^3 = \hat{C}\hat{C}\hat{C}$. Find all the eigenvalues of $\hat{C}^3$ i.e all the values of λ for which $\hat{C}^3|\phi\rangle = \lambda|\phi\rangle$.

i) Now let's invent a new operator $\hat{R}$ such that,

$$\hat{R}|1\rangle = \cos\theta|1\rangle + \sin\theta|2\rangle, \hat{R}|2\rangle = -\sin\theta|1\rangle + \cos\theta|2\rangle, \ \hat{R}|3\rangle = |3\rangle$$

Descibe in words what this operator is doing.

j) Find $|\varphi\rangle = \hat{R}|\psi\rangle$ and then compute both $\langle\varphi|\psi\rangle$ and $\langle\psi|\varphi\rangle$. Interpret your result.

k) For i, j with i and j both running over 1, 2, 3 compute the "matrix elements" of $\hat{R}$, i.e. the set of 9 numbers $R_{ij} = \langle i|\hat{R}|j\rangle$ and arrange as a matrix. Is it true that $R_{ji} = R_{ij}^*$?

l) Find the eigenvalues of $\hat{R}$.