LUMS School of Science and Engineering

Modern Physics 201 Final Examination

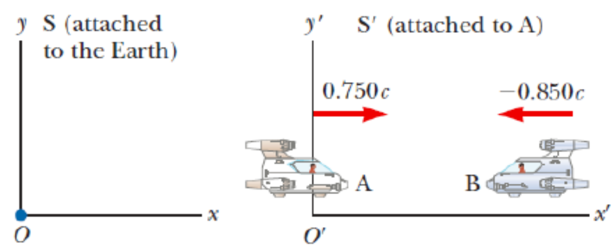
22 December 2012 Time = 3 hours

No papers/books/cell-phones/calculators are allowed.

Answer all questions.

Each question carries equal marks. All parts of a question have equal weightage

Q.1



Two spacecraft A and B are moving in opposite directions as shown. An observer on Earth measures the speed of A to be $0.75c$ and that of B to be $0.85c$. Find the velocity of B as observed on A.

Q.2 Massive stars ending their lives in supernova explosions produce the nuclei of all atoms in the bottom half of the periodic table by fusions of smaller nuclei. This problem roughly models that process. A particle of mass m moving along the x axis with a velocity component $+u$ collides head-on and sticks to a particle of mass $m/3$ moving along the x axis with velocity component $-u$.

a) What is the mass of the resulting particle?

b) Evaluate the above result in the limit $u \to 0$.

c) Explain whether the result agrees with that from non-relativistic physics.

Q.3 The Pauli exclusion principle plays a crucial role in explaining the incompressibility of matter (ref: our class discussion on white dwarfs). A measure of this incompressibility is given by the bulk modulus $B = -\dfrac{\Delta p}{\Delta V / V}$. Here Δp is a pressure change that brings about a fractional change $\Delta V / V$ in the volume of some sample of matter. For infinitesimal changes, rewrite this as $B = -V\dfrac{dp}{dV}$.

a) By considering the work done in squeezing a piece of matter, show that $p = -\dfrac{dE}{dV}$.

b) Using the result we derived in class, i.e. $E = \dfrac{3}{5} N\,\varepsilon_F$, show that $p = \dfrac{\hbar^2}{5m}\,(3\pi^2)^{2/3}\left(\dfrac{N}{V}\right)^{5/3}$.

c) Find B.

d) Give a physical reason for why even a zero temperature Fermi gas nevertheless exerts pressure on the container to which it is confined. Physically, why does the bulk modulus increase with particle density?

Q.4 Consider a beam of particles that is sent from the left on a potential barrier of height V_0.

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 \le x \le a \\ 0 & x > 0 \end{cases}$$

The energy of the particles is exactly V_0 as well. (This is a very special case, of course).

a) Write down the wavefunctions in the three different regions.

b) Solve for the reflection and transmission coefficients.

5) Suppose a proton has radius b, assumed to be very small. An electron orbits around it. Assume, for the moment, that the ground state H-atom wavefunction is exactly that which we had calculated for a point proton,

$$\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \quad (a \text{ is the Bohr radius})$$

a) What is the probability that the electron is to be found inside the proton?

b) To a good approximation you can assume ψ to be constant over the size of the proton. Show that your result becomes $P = \frac{4}{3}\pi b^3 |\psi(0)|^2$ provided that $b \ll a$.

c) Suppose that the proton is a ball of uniformly distributed charge of radius b. What is the potential inside and outside the proton?

d) Write down the Schrodinger equation for the electron's motion both inside and outside the proton. [Do not solve this. Note, however, that the wavefunction will not be exactly that which was calculated for a point proton. The difference can be calculated quite easily.]

Q.6 Consider a system whose Hamiltonian is,

$$H = \hbar\omega \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a) What are the possible energies that the system can have?

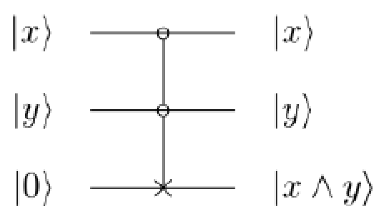
b) Suppose that the state of the system at time $t = 0$ was,

$$|\psi(t=0)\rangle = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

What is the probability of finding the system in its lowest energy state at $t = 0$?

c) Find $|\psi(t)\rangle$.

Q.7 a) What is the truth table for the quantum gate shown below?



($x \wedge y$ is true if both x and y are true)

b) The Hadamard transformation is defined as follows:

$$H: \quad |0\rangle \rightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Show that by acting with H repeatedly on an n-qubit register filled with zeros, all kets with numbers up to $2^n - 1$ are generated with equal weights,

$$H \otimes H \cdots \otimes H\,|00\cdots 0\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle$$

c) A unitary operator U is made to act upon the above state. Expalin why, or why not, the resulting state can be copied? Also explain why, or why not, the state can be teleported.

d) The 3-qubit register of a quantum computer is in the state,

$$|\Psi\rangle = \sqrt{\frac{1}{6}}\,|000\rangle + \sqrt{\frac{1}{6}}\,|010\rangle + \sqrt{\frac{2}{3}}\,|111\rangle$$

Into what state will $|\Psi\rangle$ collapse after a measurement of the third qubit? Is it entangled?